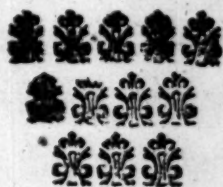


THE
Young Gentleman's
MECHANICKS:

CONTAINING

The more Useful, and Easy
ELEMENTS of *Mechanicks*,
more properly so called, of *Sta-*
ticks, and *Hydrostaticks*.

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stershire.



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By EDWARD WELLS, D.D.
Rector of Church in London

THE
PREFACE.

Nothing more needs be said, to shew the great Usefulness of the Science, denoted by the general Name of Mechanicks, than that it teaches, and explains in a geometrical Manner, those Principles, whereon is founded the Power or Force of the several Machines or Engins, made use of either in Civil or Military Affairs; and also that it explains in like manner the various Motion of heavy Bodies, consider'd either in the Air or in the Water. The bare making of Machins is indeed an illiberal or servile Employ, and fit only for Persons of the meaner Sort; whence such Handy-craft Persons are sometimes called, by way of Degradation,

The Preface.

Mechanicks. *But the Knowledge of the Principles, whereon depend mechanical Arts or Trade, is justly esteem'd a Liberal Science, not below the Study of Persons of the highest Abilities and Rank. Forasmuch as this Science both gives a very considerable Insight into the Works of Nature; (not only with respect to Sublunary Bodies, but even to the Frame, Order, and Preservation of the Celestial System;) and also may be infinitely serviceable to the Concerns of Humane Life, by the Invention of new Machines or Instruments: especially if it be cultivated by Persons, who have both Ability of Parts to invent, and also Ability of Pocket actually to experiment their Inventions, and by this Means to compleat and perfect them. For it is to be fear'd, that many an happy Invention has unfortunately miscarried, because the Inventer has not had a good Purse, as well as a good Head.*

THE

The Young Gentleman's
in the Concerns of Life, whether Civil
or Military.
By a Writer is understood in Mech-
nicks any heavy Body by a Power ()*
Whatever can move an heavy Body
and by Motion or
The Axis of Motion is that right Line
where Centres are all in the said right
Line, and the mid-point of the Axis is

THE Young Gentleman's MECHANICKS.

CHAP. I.

*Wherein are explain'd such Terms as
relate to Mechanicks in general.*

THAT Art or Science, which
teaches how to move any
given Weight, by any gi-
ven Power, we call in one
Word (†) *Mechanicks*; because it is
chiefly serviceable to the contriving of
those Machines or Engines, which are us'd

*r. Mecha-
nicks,
what, and
why so cal-
led.*

(†) The Word *Mechanicks* is deriv'd from the Greek
Word *Μηχανή*; from which is made likewise the corre-
spondent Latin Word *Machina*; and from this is form'd
our English Word *Machine*. From the same original Greek
Word is deriv'd also the Word *Mechanism*, i. e. the Arti-
ficial Contrivance of a Thing.

The Young Gentleman's

in the Concerns of Life, whether Civil or Military.

2. *A Weight, and a Power, and Motion, what in Mechanics.* By a *Weight* is understood in Mechanics any heavy Body ; by a *Power* (*), whatever can move an heavy Body ; and by *Motion*, Local Motion or the Change of Place.

3. *The Axis and Center of Motion, what.* The *Axis of Motion* is that right Line, round which a Body being mov'd, every Point of the Body describes Circles, whose Centers are all in the said right Line ; and the Mid-point of the Axis is called the *Center of Motion*. Thus Fig. 1, the Sphere or Bowl ADBE, being contriv'd to move round the right Line AB, the said AB will be the *Axis*, and the Point C the *Center of its Motion*.

4. *The Direction of Motion, what.* That right Line, wherein any Body or Power endeavours to move, is call'd the *Line of Direction*, or simply, the *Direction* of the Motion of the respective Body or Power. And if a Body moves in a Curve Line, then the Direction of its Motion in each Point of the Curve is the Tangent of the said Point. Thus Fig. 2, AB is the Direction of the Weight A, and BC the Direction of the Power at C.

(*) Hence any Weight or heavy Body is or may be esteem'd a Power, in reference to any other Weight or heavy Body, which it do's or can move.

And

And Fig. 3, the Tangents TA, TB, TC, are respectively the several Directions of a Body mov'd round the Curve ABC, at the several Points A, B, and C.

That Force, which is in Bodies mov'd, and whereby they continually move, or endeavour to move, is call'd their *Momentum*, and arises from the Weight of (or Quantity of Matter in) the Body mov'd, and the Velocity or Swiftneſs wherewith it moves.

5.
The Momentum of a Body, what.

The different Weight of Bodies arises from the different Quantity of Matter in the ſaid Bodies. Whence the Quantity of Matter in any Body may be eſtimated by its Weight. For inſtance, if a cubical Inch of Lead be ſix times heavier than a cubical Inch of Wood, it may thence be inſerr'd, that the Quantity of Matter in ſuch a Piece of Lead is ſix times as much, as the Quantity of Matter in a like Bulk of Wood.

6.
The Weight of Bodies is proportional to their Matter.

That Gravity or Weight, which arises from the more or leſs Compactneſs of Matter naturally belonging to the ſeveral Species or Sorts of Bodies, is called their *Specifick Gravity*. Thus, although a Pound of Feathers be equal in Weight to a Pound of Lead, yet Lead is ſaid to be *ſpecifically* heavier than Feathers, becauſe there is not ſo much Matter, nor conſequently ſo much Weight, in a Feather,

7.
Specifick Gravity, what.

ther, as in a Piece of Lead of the same Bulk with the Feather.

8.
Centrum
Gravium,
what.

Gravity or Weight being that, which causes in Bodies a Tendency to move (if not hinder'd by some external Force) downwards, or towards the Center of the Earth; hence the Center of the Earth is stil'd *Centrum Gravium*, or the Center to which heavy Bodies tend.

9.
The Center
of Gravity,
what.

The *Center of Gravity* in any Body is that Point, through which a Plane passing divides the Body into two Segments or Parts of equal Weight. It is this Center of Gravity, which is of chief Regard in Bodies, a Body being not properly said to descend or ascend, but when its Center of Gravity descends or ascends.

10.
Velocity,
what.

Velocity (which is the other Ingredient, that together with Weight makes up the Momentum of a Body) is that Affection of Motion, which is measur'd by comparing together the Length and Time of Motion. Thus *Equal Velocity* is that, whereby equal Length is pass'd over in equal Time: *Greater Velocity* is that, whereby either a greater Length is pass'd over in equal Time, or an equal Length in less Time: *Less Velocity*, when contrariwise. Further, a Body that moves an hundred Yards, may be said to move with *double* the Velocity of a Body, that moves

moves but fifty Yards in the same Time; and with *triple* or thrice the Velocity of another Body, that moves but thirty-three Yards and one third in the same Time; and with *quadruple* or four-times the Velocity of a Body that moves but twenty-five Yards in the same Time. And so on.

The Momentum of a Body, arising (as has been observ'd §. 5,) from its Weight and Velocity, may be known by multiplying one of its Ingredients into the other. And consequently the different Momentums of different Bodies may be known by multiplying the Weight of each Body into its respective Velocity. Thus suppose the Weight of a Body A to be ten Pound, and of any other Body B to be four Pound; and the Velocity of B to be five times as much as that of A. The Momentum of A will be 10; viz. 10 Pound, the Weight of A, being multiplied into 1 for its Velocity, (the Velocity of A to B being suppos'd as 1 to 5,) the Product will be 10 for the Momentum of A. And in like manner, 4 Pound, the Weight of B, being multiply'd into 5, as the Velocity of B, (for the Velocity of B to A is suppos'd as 5 to 1) the Product will be 20 for the Momentum of B. Whence it appears that the Momentum of B is

II.
The Momentum of a Body, how to be known.

12.
The Fundamental
Theorem
of Mechanics.

B is double to that of A, being as 20 to 10. Hence is deduced the following Theorem, which may be stiled the Fundamental Theorem of Mechanics, viz. *When Bodies have their Velocities reciprocally proportional to their Weights, their Momentums will be equal.* Let the Weight of a great Body A be 500 Pound, and the Weight of a little Body B be 50 Pound; and let their respective Velocities be reciprocally proportional, (i. e. as the Weight of A is to that of B as 10 to 1, so let the Velocity of B be to that of A as 10 to 1.) It will hence follow, that the Momentum of B will be just equal to that of A. Namely, 500 Weight $\times$ 1 Velocity = 500 the Momentum of A; and 50 Weight $\times$ 10 Velocity = 500 the Momentum of B.

13.
The general End
and Effect
of Mechanical
Engin's.

Hence it appears, not only that any little *may have* its Momentum equal to the Momentum of any great Body which is mov'd with a given Velocity; but also what Method is to be taken. in order to make it *actually have* so; namely, by contriving that the Velocity of the Power may be so much greater than the Velocity of the Body to be mov'd, as the Weight of the Body is greater than the Weight (or somewhat equivalent thereto) of the Power. On which Account it is, that the foremention'd Theorem may be

be justly esteem'd the Fundamental Theorem of Mechanicks; forasmuch as thereon is founded the principal Mechanism or Contrivance of Machin's or mechanical Engin's, made use of to draw or raise heavy Bodies; the whole End and Effect of these being to give such a Velocity to the Power, or (which comes to the same) so to diminish the Velocity of the Weight or heavy Body, as that the Momentum of the Power may be equal, or somewhat exceed, the Momentum of the Body so to be drawn or rais'd.

And this is what will be illustrated in the six following Chapters, in reference to the six simple and therefore primary Machin's, or (as they are otherwise called) *Mechanical Powers*; to some one or more of which may be reduc'd all other mechanical Engin's. And I shall speak of them in the Order, wherein they here follow (*) reckon'd up, viz.

14.

The simple
mechanical
Powers, six
in Number.

I Vēctis

(*) Whereas some reckon but five simple Machines or mechanical Powers, comprehending the Balance under the Leaver; and others that reckon six simple Machines, usually reckon the Balance as the first, and the Leaver as the second; I have judg'd it more proper to agree with the latter, in reckoning six simple or primary Machines, and to fall in so far with the former, as to reckon the Leaver as the first, and the Balance as the second simple Machine, inasmuch as the Leaver is a more simple Machine than the Balance;

1. *Vestis* or the *Leaver*.
2. *Libra* or the *Balance*.
3. *Axis in Peritrochia* or the *Axle in the Wheel*.
4. *Trochlea* or the *Pulley*.
5. *Cochlea* or the *Screw*.
6. *Cuneus* or the *Wedge*.

Balance; and the Balance is no other than a particular Kind of Leaver. In like manner, whereas some reckon the Wedge as the fifth, and the Screw as the sixth simple Machine, I have rather follow'd them, who have inverted the said Order, forasmuch as the Screw partakes more of the Faculty of the Leaver than the Wedge.

C H A P. II.

Of the Leaver.

I.
A Leaver,
what.

THE *Leaver*. is so call'd from the principal Use thereof, which is *levare*, i. e. to lift or raise up heavy Bodies. For the more easy explaining the Mechanism thereof, it is here conceiv'd as a Line, which will not bend, and may be turn'd round about an unmoveable Point; which is therefore the Center of its

its Motion, and rests upon somewhat thence call'd the *Fulcrum*, i. e. Prop of the Leaver. Thus Fig. 4, AB is the Leaver, C the Center of its Motion, and F the Fulcrum or Prop.

The main Doctrine of the Leaver may be comprised under two Theorems, where-
 of the first is this: *The Space (or Arch) describ'd by each Point of a Leaver, and consequently the Velocity of each Point of a Leaver, is, as its Distance from the Fulcrum or Prop.* For instance, the Leaver AB (Fig. 4,) being mov'd from ACB to aCb, the Point A will describe the Arch Aa, and the Point B will describe the Arch Bb; and by reason of the like Sectors ACa and BCb, Aa is to Bb as AC to BC, that is, the Spaces describ'd by the Points A and B are as their Distances from C the Center of Motion, or (which comes to the same) from F the Fulcrum. Now Powers being apply'd to the Points A and B, which draw the Arms CA and CB of the Leaver AB perpendicularly, the Spaces describ'd by them will be (not the Arches Aa and Bb, but (the Perpendiculars AD and EB, these being the Measures of the Progress made by the Powers according to their respective Direction. But the Triangles ACD and BCE being equiangular, therefore
 (*) AD

2.

Theorem
the first.

(*) AD is to BE as AC to BC or aC to bC , that is, the Lengths gone over by the Powers according to their respective Directions, are as their Distances from C the Center of Motion, or (which comes to the same) from F the Fulcrum. Now Velocity being (according to Chap. 1. §. 10) that Affection of Motion, which is measur'd by comparing together Length and Time, and consequently (one Sort of) greater Velocity being that, whereby a greater Length is past over in equal Time, as in the Case before us; hence it follows, that the Velocity either of the Point B, or of the Power apply'd thereto, is so much greater than the Velocity of the Point A or the Power apply'd thereto; (as the Arch Bb or the Line BE is greater than the Arch Aa or the Line AD; and consequently so much greater) as BC or bC is than AC or aC ; that is, the Velocity of each Point in the Leaver, or of any Power apply'd thereto, is as its Distance from (C the Center of Motion, or from) F the Fulcrum (†).

(*) By Theorem 9, of the Young Gentleman's Geometry.

(†) For the Sake of such as may not readily apprehend the foregoing Geometrical Demonstration of the first Theorem, the same is sensibly demonstrated in Fig. 4, by actually dividing BC, bC , Bb , and BE into six equal Parts, each of the same Length or Compass as AC, aC , Aa , and AD respectively.

The

The Truth of the first Theorem being demonstrated and apprehended, it will be easy to apprehend the Truth of the second Theorem, which is this: *In a Leaver the Power, which is to the Weight (or Body) to be rais'd, as the Distance of the Weight (according to its (||) Line of Direction) from the Fulcrum is to the Distance of the Power (according to its Line of Direction) from the Fulcrum, will counterpoise or hold up the Weight; and therefore being a little increas'd, will raise the Weight.* For it has been shewn under Theorem 1, that the Spaces describ'd by (the Powers apply'd to each End of the Leaver, i. e. by) the Power apply'd to one End and the Weight apply'd to the other, are proportional to their said Distances from the Fulcrum; and consequently that their Velocities, being proportional to the said Squares, are also proportional to the said Distances. Wherefore if (Fig. 5) CA the Distance of the Weight from the Fulcrum be to CB the Distance of the Power from the Fulcrum, as the Power P is to the Weight W, it will follow, that likewise the Velocity of the Weight

(||) Thus Fig. 5, Bb the Line of Direction of the Power is equidistant from C to Aa, the Line of the Weight's Direction.

will

will be to the Velocity of the Power, as the Power is to the Weight. And consequently the Momentum of the Power will (by the Fundamental Theorem Chap. 1. §. 12) be equal to the Momentum of that Weight; and therefore the Power will be able to sustain the Weight; and if it be but a little increas'd, (either by the Addition of some new Force, or else by increasing its Distance from the Fulcrum) it will be able to raise the Weight.

4.
Theorem
the second
illustrated
by an Ex-
ample.

An Example will illustrate the Matter. Let *W* the Weight of the Body to be rais'd be 120 Pound, and the Force of the given Power *P* be equivalent only to 20 Pound, and consequently the Weight be to the Power as 6 to 1. Wherefore *P* the Power being so plac'd at *B* (*Fig. 5*) and *W* the Weight at *A*, as that *BC* may be six times as long as *AC*, in such a Position the Power will have a Force sufficient to counterpoise, and so hold up the Weights. For the Velocity of *P* to that of *W* being proportional to their Distances from *C*, will be in this Case as 6 to 1; and consequently the Momentum of *P*, viz. 20 Pound $\times$ 6 Velocity = 120 = 120 Pound $\times$ 1 Velocity, the Momentum of *W*. Wherefore the Power *P* (having a little new Force added to its own, or) being remov'd a little farther distant from

from C, viz. to *b*, and so having its Velocity a little more increas'd, it will raise *W* the Weight. For supposing *bC* to be to *AC* as 7 to 1, the Momentum of *P* will be 20 Pound $\times$ 7 Velocity = 140; which is considerably greater than the Momentum of *W* at *A*, this being (as afore found) but 120.

Having thus shewn the Truth of the two foregoing Theorems, and these being applicable to the other following mechanical Powers, as well as to other Cases in respect of the Leaver, it may be of Use to observe here, that the Import of the second Theorem here laid down will for the future be express'd in short thus; $P : W :: CW : CP$, that is, As much as the Force of the Power *P* is less than the Weight *W*, so much less must be *CW* the Distance of the Weight from the Fulcrum, than *CP* the Distance of the Power from the Fulcrum.

And as this has been shewn to hold good in respect of the Leaver (as it is called by some) of the first Kind, that is, when the Fulcrum is between the Power and the Weight; so does it hold good also in respect of the Leaver of the second Kind, when the Weight is between the Power and the Fulcrum; and also in the Leaver of the third Kind, when the Power is between the Weight and the Ful-

(B)

crum.

5.

Theorem
the 2d ex-
press'd in
short by
Letters.

6.

The Kinds
of Leavers.

crum. For in a Leaver of the second Kind, the Weight W at D (*Fig. 6.*) has (as is obvious) just the same Effect, as it would have on the other Side of the Fulcrum and at the same Distance from it, *viz.* at E . And in like manner the Power P at D (*Fig. 7.*) has just the same Effect, as it would have at E , CE being equal to CD . Wherefore the Leaver of the second and third Kind being thus in effect no other than that of the first, the Mechanism of these depends on the same Theorem or Rule, as the Mechanism of that, *viz.* $P : W : CW : CP$.

7.
Scissors,
Pinchers,
&c. Kinds
of Leavers.

To the Leaver of the first Kind are referr'd Scissors, Pinchers, Snuffers, and such-like Instruments, they consisting as it were of two such Leavers join'd together, but acting contrary Ways; as may be easily apprehended by *Fig. 8* and *9*. To the Leaver of the second Kind are referr'd the Oars of a Boat, as also such Cutting-Knives as are fix'd at one End. To the Leaver of the third Kind may be referr'd a Ladder, lifted up between the two Ends, in order to rear it against a Wall. Lastly, an Hammer drawing a Nail is referr'd to a fourth Kind of Leaver, call'd a *Bended* Leaver. For the Nail is as the Weight at one End A , and the Hand of him that draws the Nail is as the Power at the other End B of the Leaver.

ver ; and the Point C (*Fig. 10*) is as the Fulcrum. And this fourth Kind of Leaver is likewise reducible to the First, the Weight or Nail at A having the same Effect, as if it were at *a*, *Fig. 10*.

As a Weight carried on a Pole or Stick between two Persons may also be referr'd to a Leaver of the second Kind, so it may be of good Use to observe, that agreeably to the foremention'd Theorem P : W :: CW : CP, the Weight may be so plac'd, as that the Burden thereof shall be proportion'd to the different Strengths of the two Persons ; Namely, by taking P to denote the Strongest, W the Weakest, and C the Weight. For then the Import of the said Theorem will be this : As much as the Strength of the Strongest P is greater than the Strength of the Weakest W, so much must CW the Distance of the Weight from the Weakest, be greater than CP the Distance of the Weight from the Strongest. For Example, suppose the Leaver or Stick PW (*Fig. 11*) to be divided into 18 equal Parts, and the Person at P to be twice as strong as the other at W, then C the Body to be carried must be plac'd at 6. For then the Distance CW will contain 12 of the equal Parts, into which the Stick is divided, and the Distance CP will contain but 6 of them ; and so $CW = 2CP$,

(B 2)

and

8.

How to place a Weight or Body on a Stick or the like, so as to make the Burden proportional to the different Strengths of them that carry the Stick.

The Young Gentleman's

and consequently the weakest Person at W will sustain but half so much as the strongest at P. In like manner, if the Body c were placed at 3, then the Person at P would sustain 5 times so much, as the other at W; because then the Disproportion of the Distances would be as 15 to 3, cW containing 15 of the equal Parts, whereof cP would contain but 3. Lastly, if the Body c were placed at 9, that is, in the Mid-point of the Stick or Pole, then each Person P and W would bear equal Burden, because the Distance $9P = \text{Distance } 9W$.

9.
The like
explain'd
as to draw-
ing any
Weight.

And the same Theorem holds good in respect of Drawing (as well as Carrying) any Burden. For (Fig. 12) let WC represent the Pole of any Waggon or Coach, and AB the Cross-bar, at whose Extremities are Spring-trees DE and FG, to which are to be fasten'd the Horses or the like. Because A and B are equidistant from C, therefore the Horses at A and B will draw equal Weight. Whereas if one of the Spring-trees be fasten'd at 1, then the said Horse would draw a fourth Part more of the Weight W, than the other Horse at B; forasmuch as the Distance C1 is a fourth Part less than the Distance CB. And so if one Horse was fasten'd to 2, he would draw twice as much as the other at B; because C2 is

is but the Half of CB. And so on.

By the Mechanism of the Leaver may we also account for the Reason, why it is more difficult to lift up a Staff (or the like) of any Length, at or near one of its Ends, than nearer the Middle: Namely, because the Momentum of any Thing so sustain'd increases proportionably to its Distance from its Fulcrum or Prop. And the same is the Reason, why it is more difficult to lift up a long Staff or the like, at Arm's Length, or keeping the whole Arm from Hand to Shoulder extended out, than when we bend the Arm at the Elbow. Namely, because in the former Case the Shoulder is the Fulcrum, or Prop, in the latter Case the Elbow; and therefore in the former the Distance of the Weight from the Prop, is greater than in the latter. And so much for the Leaver.

10.

Other Particulars solv'd.

C H A P. III.

Of the Balance.

1.
The Beam
of a Ba-
lance no o-
ther than
a Sort of
Leaver.

THE Beam AB (Fig. 13 and 14) is the principal Part of a Balance, and is no other than a Leaver of the first Kind, which (instead of resting on a Fulcrum at C the Center of its Motion) is held up from above by somewhat fastened to C its Center of Motion.

2.
The Me-
chanism
of the Ba-
lance ex-
plain'd in
general.

Hence the Mechanism of the Balance depends on the same Theorem as that of the Leaver, viz. $P : W :: CW : CP$. Where, taking P to denote the known Weight, W the unknown Weight, and C the Center of the Balance's Motion, the Import of the Theorem is this: *As much as the known Weight is less than the unknown Weight, (i. e. the Weight sought of the Body weigh'd) so much will the Distance of the unknown Weight from the Center of Motion be less than the Distance of the known Weight from the Center of Motion, where the said two Weights will counterpoise one the other; and consequently the known Weight shew the Quantity of the unknown Weight.*

For

For Instance, on the (*) Roman Balance or Steel-yard, (*Fig. 13*) the unknown Weight or Body to be weigh'd being apply'd to A, and the known Weight P being mov'd up and down, nearer and farther (from the Point C) upon the Arm BC, and being found at the Distance 5 to counterpoise W the unknown Weight, thence it follows, (*viz.* because the Distance CP is 5 times greater than the Distance CW, or which is the same, CA) that the Quantity of the unknown Weight W is 5 times as much as the Quantity of P the known Weight. And therefore, supposing P to be one Pound Weight, W will be five Pound Weight; or supposing P to be two or three Pound Weight, W will be respectively ten or fifteen Pound Weight.

3.
The Mechanism of the Roman Balance; or Steel-yard particularly explain'd.

But now, because in a common Balance or a (†) Pair of Scales, (*Fig. 14*) the two Arms CA and CB of the Beam AB are (or ought to be) always of an equal Length; hence the Quantity of the

4.
The Mechanism of the common Balance or of a Pair of Scales, explain'd.

(*) This Sort of Balance is call'd the *Roman Balance*, from its being much us'd at *Rome*. And it is commonly call'd a *Steel-yard*, as being a Sort of Yard made of Steel.

(†) So the Latins call this *Libra* or Balance by the peculiar Name of *Bilanz* (namely *a binis Lancibus*, i. e. from the two Scales belonging thereto) whence is made our English Word *Balance*.

(B 4)

known

known Weight P in one Scale must in it self be equal to the Quantity of the unknown Weight W put in the other Scale. And therefore the Quantity of the unknown Weight on this Sort of Balance is found out, (not as on the Steel-Yard, by moving *one* and the same Weight nearer or farther from C the Center of Motion, but) by putting in *various* Weights of known Quantity, 'till the known Weight counterpoises the unknown; and thereby shews the Quantity of the unknown Weight.

5.
Bare
Weight is
strictly true
Weight.

For it is to be observed, that the known Weight then shews the exactly *true* Quantity of the unknown Weight, when the one counterpoises the other. For the usual Custom of making the Weight of the Thing somewhat *more* than counterpoise the known Weight by which it is weigh'd, (even so as to bring the Scale, wherein the Thing bought is put, to the Ground or the like) has arose from Tradesmen being willing to get Customers, by allowing them somewhat more than strictly true, or (as it is commonly and (*) properly enough call'd) *bare* Weight. And consequently, when we say of a

(*) Because hereby is denoted true Weight, consider'd barely as to Weight, without any respect to Custom, &c.

Tradesman, who is more free than ordinary in what he allows over bare Weight, that he makes *good* Weight, or *better* Weight than another, that makes only bare Weight ; thereby is to be understood, not that the Weight he makes is true, or truer than the other's, in respect of its intrinsick Quantity ; but that it is good, or better than the other's, in respect of the Advantage accruing to the Buyer, by what is allow'd over bare or true Weight.

It is also not to be omitted, that as one Body is said to counterpoise (or to be of equal Weight with) another, when they being severally apply'd to the two different Arms of the Balance, the Beam lies (†) horizontally or equally level ; so this is call'd in our common Speech (not only bare Weight for the Reason afore assign'd, but also) *standing* Weight, forasmuch as in this Case the Beam stands even, neither Arm ascending or descending more than the other ; and for the same Reason the Bodies in this Case are said by the Learned to be in *Æquilibrium*.

6.

Standing Weight, and to be in *Æquilibrium*, what.

(†) This is shewn by the Piece of Iron CE, call'd the Tongue, being in a Line or directly between TX and TS.

Lastly,

7. *How to discover a false Balance or Pair of Scales.* Lastly, it is well to be observ'd, that the Truth or Exactness of (a common Balance or) a Pair of Scales greatly depending on its two Arms being of equal Length, as well as of equal Weight in themselves, hence it follows, that if one Arm be longer than the other, it will be a false Balance, or not shew the true Quantity of the unknown Weight, or (which is the same) the true Weight of the Thing weigh'd. For instance, suppose *Fig. 14*, the Arm AC to be divided into ten equal Parts, and the other Arm BC to contain but nine such other Parts; and consequently AC to be longer than BC by a tenth Part. It will follow by the Fundamental Theorem (*Chap. 1. §. 12*, or which comes to the same, by this Theorem, $P : W :: CW : CP$.) that a known Weight of nine Pound apply'd to A, will have its Momentum equal to a Body of ten Pound Weight apply'd to B; for $9 \text{ Pound} \times 10 \text{ Velocity} = 90 \text{ Momentum} = 10 \text{ Pound} \times 9 \text{ Velocity}$.) and consequently will counterpoise one the other in such a false Balance, though their Weights be not equal in themselves. The Way to discover such false Balances, is by changing the known and unknown Weight into the contrary Scales. For upon such a Change, the Inequality of the Weights will manifestly

ly (*) appear ; whereas in a true Balance or Pair of Scales, such a Change of the Scales will make no Difference, there being no Difference in the Length of the Arms, and consequently no Difference in the Velocity of the known or unknown Weight, to which ever Arm they be apply'd.

(*) Namely, the Body being of 10 Pound Weight in it self, and when apply'd to A, its Velocity being in respect to the Velocity of the known Weight of 9 Pound in it self, as 10 to 9, the Momentum of the said Body will be (10 Pound * 10 Velocity =) 100 ; whereas the Momentum of the known Weight will be (9 Pound * 9 Velocity =) 81. And consequently the known Weight will not counterpoise the Body by a great deal put in the Scale of the shorter Arm BC, though it did so in the Scale of the longer Arm AC.

C H A P. IV.

Of the Axle in the Wheel.

THIS Machine is so called, as consisting principally of a Wheel BGHI, and a Cylinder or Axle ADEF. About which Axle goes the Rope, which draws the Weight W, when the Axle is turn'd round by means of the Wheel ; as Fig. 15.

I.
This Machine, why so called.

As

2.
It is in effect no other than a Leaver, capable of being perpetually turn'd round its Fulcrum.

As the Balance is no other than a Leaver, particularly apply'd to finding out the Quantity of an unknown Weight, so this mechanical Power is no other than a Leaver, apply'd to the same Use as other Leavers are, (*viz.* to the raising up of great Weights) with this particular Advantage added thereunto: Namely, whereas in the Leaver it self the Motion can be continu'd only for so short a Space *Aa*, (*Fig. 4 and 5*) as is answerable to the little Distance *CA* between the Fulcrum and the Weight; in this Invention that Inconvenience is remedy'd, forasmuch as by continual turning round the Wheel, and so (together with it) the Axle, the Weight may be rais'd to any Height, as Occasion shall require.

3.
Its Mechanism explain'd.

As this Machine is no other than a Leaver *AB* capable of being perpetually turn'd round, so the Fulcrum of it or Center of its Motion is *C*, and the End *B* is that to which the Power *P* is to be apply'd, and *A* the other End, to which is to be apply'd the Weight *W* to be raised. Hence the Mechanism of this Engine depends on the same Theorem, as that of the Leaver, *viz.* $P : W :: CW : CP$; that is with particular reference to this Machine; If as much as the Power is less in it self than the Weight, so much the Radius of the Axle be less than the Radius

Radius of the Wheel, (and consequently so much the Circumference of the Axle be less than the Circumference of the Wheel; and consequently so much the (†) Velocity of the Weight apply'd to the Axle be less than the Velocity of the Power apply'd to the Wheel) by the Fundamental Rule the Momentum of the Power will be equal to that of the Weight, and consequently the Power will counterpoise and so hold up the Weight; and therefore the Power being a little increas'd will raise the Weight.

The Momentum of the Power thus increasing according to the Disproportion between the Circumference of the Wheel, and the Circumference of the Axle; hence it follows, that the Line of the Power's Direction must always touch the Circumference of the Wheel, so as to make a(*)right Angle with the Ray, that so the same Distance between the Power and the Center of Motion, and consequently the same Momentum of the Power, may be every where alike preserv'd ;

4.
An Observation.

(†) For the Weight, as is evident from the Make of this Machine, is rais'd up no more than is answerable to the Length of the Rope that goes round the Axle in one turning round of the Wheel.

(*) That is, so as to be the Tangent of the Point of the Circumference it touches, according to Chap. 1. S. 4.

which

which otherwise would not be so. For supposing (*Fig. 16*) the Power P to be so apply'd to any Spoke, *viz.* G , as that the Line of its Direction be GH , then the Distance of the Power is to be esteem'd IC , and so less than BC , and consequently the Momentum of the Power will be so much lessen'd, as IC is less than BC . Whereas if the Line of the Power's Direction were CH , then the Momentum of the Power at C would be the same as at H .

5.
Machines
and Instru-
ments refe-
rable
thereunto.

To this third simple Machine are re-
ferr'd the Crane *Fig. 17*, and all Engines
which consist of Wheels with Teeth, as
Fig. 18, and also the Machine represent-
ed *Fig. 19*, and the several other Instru-
ments represented *Fig. 20, 21, 22, 23,*
24 and 25. In each of which A repre-
sents the Place of the Weight, B the Place
of the Power, and C the Center of Moti-
on or Fulcrum.

CHAP.

CHAP. V.

Of the Pulley.

THE *Pulley* is a Machine consisting of one or more little Wheels, about which a Rope being put and (*) pull'd at one End, the said Rope makes the said Wheels turn round upon their proper Axles, and thereby at the same time moves the Weight or Body to be drawn or rais'd, as *Fig. 26, 27, 28.*

1.
A Pulley, what.

If the Pulley it self be fasten'd up to some Place or Mark H, and the Weight W be apply'd to one End of the Rope, and the Power P to the other, (as *Fig. 26,*) then it is evident, that as much as the Weight W ascends, just so much the Power P descends; and consequently that they both move with the same Velocity. Wherefore in this Case the Force of the Power must in it self be equivalent to the Weight of the Body rais'd, in order to

2.
An upper Pulley, what, and its Use explain'd.

(*) The Effect of this Machine being thus produc'd by pulling the Rope, hence it takes the Name of the *Pulley* among us.

make

make its Momentum equal to the Momentum of the Body. And hence it follows, that such a Pulley does not conduce to the increasing of the Momentum of the Power, or (which comes to the same) to the decreasing the Momentum of the Body or Weight to be rais'd; but only to the keeping of the Cords from fretting; and so to the making them move more easily than otherwise they would. And, because when there is Occasion to apply several Pulleys together, such as are of this Sort, (or which the Ropes go over,) are plac'd uppermost; hence these are distinguish'd by the Name of *Upper Pulleys*.

3.
A lower
Pulley,
what, and
its Use ex-
plain'd.

But if (not the Pulley it self, but) one End of the Rope be fasten'd to an Hook H or the like, and the other End be pulled by the Power P, and the Weight W hangs on to the Pulley, (as *Fig. 27.*) then it is evident, that in order to raise the Weight W one Foot, each Part AH and Bb of the Rope must be shorten'd one Foot, (reckoning downwards from the Hook H and its opposite Point b) and consequently that in the same Time the Power P must move two Foot. Wherefore in this Case the Velocity of the Power will be double to that of the Weight; and consequently if the Power be subduple to the Weight, that is, as 1 to 2, its

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its Momentum will be equal to that of the Weight, and so will sustain the Weight. And therefore if the Power be a little increased, it will be able to raise the Weight. And the same holds when there are two Wheels or Pulleys, one Upper and one Lower, as *Fig. 28.*

If (as *Fig. 29*) there be three Wheels, and so the Rope goes from Wheel to Wheel three times, *viz.* from A to B, and from C to D, and from E to F; and consequently in order to raise the Weight W one Foot, each of the three Parts AB, CD, and EF of the Rope must be shorten'd one Foot, which can't be done without the Power P moving forward three Foot in the same time; in this Case the Velocity of the Power will be triple or thrice as much, as the Velocity of the Weight. And therefore, though the Power be to the Weight but as 1 to 3, yet its Momentum will be equal to the Momentum of the Weight, and so will sustain it; and if the Power be but a little increased, will raise the Weight.

4.

Further of the same.

In like manner it is evident from *Fig. 30*, that if there be four Wheels, and so the Rope goes four times from Wheel to Wheel; then, while the Weight W is rais'd one Foot, the Power P will move forward four Foot; and consequently

5.

The same yet further exemplify'd.

(C)

(by

(by the fundamental Theorem) a Power P, which is to the Weight W but as 1 to 4, will have its Momentum equal to the Momentum of the Weight ; and therefore being a little increas'd, will raise it.

6. Hence the Mechanism of the Pulley depends on this universal Theorem, viz. *The general Theorem relating to the Pulley.* If a Power is to the Weight, as 1 to the Number of the Ropes (*i.e.* of the Parts of the Rope, viz. AH and Bh, Fig. 27, and 28 ; and AB, CD, EF, Fig. 29, and AB, CD, EF, GH, Fig. 30,) apply'd to the lower Pulleys, then the Power will be able to hold up the Weight ; and therefore being a little increas'd, will raise the Weight.

7. For Instance, Suppose the Weight to be rais'd 360 Pound, and the Power in it self to be equivalent only to 180 Pound, (*i.e.* the Power to be to the Weight as 1 to 2,) therefore one lower Pulley (either with, as Fig. or without an upper Pulley) will render the Momentum of the Power equal to that of the Weight, namely, by rend'ring the Velocity of the Power double to that of the Weight. For $180 \text{ p.} \times 2 \text{ Velocity} = 360 \text{ Momentum} = 360 \text{ p.} \times 1 \text{ Velocity}$. If the Power be to the Weight as 1 to 3, or equivalent only to 120 Pound, then there must be three Wheels, that so the Velocity

city of the Power may be triple to that of the Weight; Namely, $120 p. \times 3$ Velocity $= 360$ Momentum $= 360 p. \times 1$ Velocity. And so on.

CHAP. VI.

Of the Screw.

THE Screw, which makes here the fifth simple mechanical Power, is a Right Cylinder, having several Risings and Hollows alternately winding round it in a spiral manner. It consists properly of two Parts, one call'd the *outward* Screw, the other the *inward*; namely, because the former receives the latter into it. Whence the inward Screw is otherwise call'd the *Male*, and the outward the *Female* Screw. The Male or inward is denoted *Fig. 31*, by *MS*, the Female or outward by *TS*.

This Machine is chiefly us'd to squeeze or press Things, and by that Means, in some Cases, to break Things. It is also sometimes used to raise up any Weight, as to thrust it forward.

1.
The Screw,
what's

2.
Its Uses.

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For

3.
This in
some sort a
compound
Machine.

For the more easie Use of this Machine, (*i. e.* for the more easie moving the Male Screw through the Female, or the Female along the Male) it is usual to apply thereto an Handle or round Stick, AB; and so to the Power of the Screw to adjoin also the Power of the Axle with the Wheel: The Handle or Stick AB by its turning round, supplying the Place of a Wheel, and the Cylinder of the Screw being as the Axle to the said Wheel. So that this Machine is in some sort a compound Machine.

4.
The funda-
mental
Theorem
particular-
ly apply'd
to this Ma-
chine.

Now the Effect of this Machine depends upon the same fundamental Theorem, as the other; which fundamental Theorem particularly apply'd to the Fabric or Make of this Machine, stands thus: *If, as the Compass describ'd by the Power in one Turn of the Screw, is to the Interval or Distance between any two immediate spiral Windings (measur'd according to the Length of the Screw) so is the Weight or Resistance to the Power; then the Power and the Resistance will be equivalent one to the other, and consequently the Power being a little increased will move the Resistance.*

5.
The same
further il-
lustrated or
prov'd.

For it is evident from the bare Inspection of this Machine, that in one Turn of the Screw, the Weight P (*Fig. 33*) is so much lifted up, or the Resistance O (*Fig.*

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Fig. 31 and 32) is so much remov'd, 'or the Thing to be press'd is squeez'd so much closer together, as is EG the Distance between two immediate Spirals; and in the same Time the Power (apply'd to A or B) to be mov'd so much, as is the Compass AB, describ'd by the said Power in one Turn of the Screw. Wherefore the Velocity of the Weight (or whatever answers thereto) will be to the Velocity of the Power, as is the said Distance between the Spirals to the Compass describ'd by the Power, in one Revolution or Turning round of the Screw. And therefore by the fundamental Theorem, if as the Power is to the Weight, so the Distance between the Spirals to the Compass describ'd by the Power, the Power will have its Moment equal to the Moment of the Weight or Resistance. And therefore the Power, being a little increas'd, will overcome the Resistance.

C H A P. VII.

Of the Wedge.

I.
A Definition
of a
Wedge,
and its se-
veral
Parts.

THE Wedge is commonly made of Iron or some harder Sort of Matter, in Shape of a Prism not very high, whose opposite Bases are Isoscelar Triangles. The Height of either of these Triangles is esteem'd the Height of the Wedge, and the Basis of either Triangle is esteem'd the Thickness of the Wedge; and the right Line which joins together the Vertexes of both the Triangles is call'd the Edge of the Wedge; and the Parallelogram which joins together the Bases of the two Triangles, is call'd the Back of the Wedge.

2.
Its Effect
accounted
for by the
fundamen-
tal Theo-
rem.

The Effect of this Machine depends likewise upon the same fundamental Theorem as the former; which being particularly apply'd hereto will stand thus: *If the Power directly apply'd to the Back of the Wedge be to the Resistance to be overcome by the Wedge, as the Thickness of the Wedge is to its Height; then the said Power will be equivalent to the Resistance; and therefore being increas'd will overcome it.*

That

That Tenaciousness or Firmness, where-
by the Parts of the Wood (or the like) adhere one to the other, is the Resistance to be overcome by the Wedge. Now it is evident, that while the Wedge is drove into the Wood, the Way or Length which it has gone according to its own Propensity, is (*Fig. 34*) BA; and in like manner DC is the Way or Length gone in the same Time by the Impediment, *i. e.* the Parts C and D of the Wood are so far divided asunder. And accordingly as the Wedge is drove down further and further along its Height, so the Parts C and D of the Wood are divided more and more along the Thickness of the Wedge; and in the whole Progress proportionably, as is evident from the Nature of a Triangle. Whence it comes to pass, that if as the Thickness of the Wedge (*i. e.* the Way of the Impediment, and consequently its Velocity) is to the Height of the Wedge (*i. e.* the Way, and consequently the Velocity of the Power,) so the Power to the Impediment or Resistance, then the Momentum of the Power and the Impediment will be equal the one to the other; and consequently the Power being increas'd will overcome the Resistance,

4.
Other Instruments
reducible to
the Wedge.

To the Wedge may be referr'd all Edge-Tools, and Tools that have a sharp Point, in order to cut, cleave, slit, chop, pierce, bore, or the like: As Knives, Hatchets, Swords, Bodkins. &c. And thus I have gone through the six simple Machines, or primary mechanical Powers, so far forth as seems requisite to the Design of this Treatise.

5.
Mechanicks
what, in the
more re-
strain'd
Sense of
the Word.

It remains only to observe here, that as the Word *Mechanicks* in its largest Acceptation comprehends the whole Doctrine or Science of Local Motion; so in a restrain'd Sense it is taken particularly to denote the Doctrine of Motion, as apply'd to Machines or mechanical Engines, the more simple of which we have here describ'd.

C H A P.

C H A P. VIII.

Of Staticks.

THE Word *Staticks* denotes in the Greek Language the Science or Doctrine of Weights, and consequently in its largest Acceptation, is of the same Extent with *Mechanicks* in the largest Sense thereof, both comprehending the whole Doctrine or Science of Motion with reference to the Weights of Bodies; and so being synonymous Words, or Words of the same Import. But as the Word *Mechanicks* in a restrain'd Sense is taken to denote more particularly the Doctrine of Motion and Weights in reference to Mechanical Engines; so the Word *Staticks* in a restrain'd Sense is taken to denote more especially that Part of the Doctrine of Motion, which arises from the Weight of natural Bodies consider'd in themselves or compar'd together, and which consequently has not at least so immediate a respect to mechanical Engines. In this Chapter therefore I shall take notice of such Particulars as relate to *Staticks* in its restrain'd Sense, and are agreeable to the Design of this Treatise.

I.
Staticks,
what.

It

2.
The Motion
of heavy
Bodies
down-
wards is
uniformly
accelera-
ted.

It is then observable in the first Place, that the natural Motion of heavy Bodies (*i. e.* That Motion of them downwards, which arises from the natural Principle of their Gravity or Weight) is found by Experience to be a *Motion uniformly accelerated*, *i. e.* a Motion which acquires in equal Times equal Degrees of Velocity. For when an heavy Body descends, the same Gravity, which gave it a Motion downwards at first, will all along continue to act alike upon it, and so to give it in the second Instant of Time a Motion equal to what it gave it at the first, and again in the third Instant or Minute to give it likewise a Motion equal to that at first, So that at the third Minute the Sum of the whole Motion will be the Triple of the first; and so at the fourth Minute the Sum of the whole Motion will be Quadruple to what it was at the first. And so the Degrees of Motion, or Velocity of the Body moved, will always increase, as does the Time wherein it moves.

3.
The Spaces
thro' which
Bodies fall,
are as the
Squares of
the Time
wherein
they fall.

Hence it comes to pass, that the Spaces, through which Bodies descend in their Fall, are as the Squares of the Times they take up in Falling, always accounting from the Beginning of the Fall. For Instance, if a Body be ten Minutes a falling, and the first Minute it falls a Mile, the second Minute it will have fall'n four Miles,

Miles, and the third Minute nine Miles; and so on: According to which Proportion at the tenth Minute, the Body will have fall'n an hundred Miles. Whence it appears, that if the Times be taken in the arithmetical Progression of natural Numbers, viz. 1, 2, 3, 4, 5, &c. the Spaces describ'd in those Times, reckoning from the Beginning of the Motion, will be as 1, 4, 9, 16, 25, &c. This is wont to be demonstrated or at least illustrated, by considering (*Fig. 35.*) AB to represent the first equal Part of Time, and BC to represent the first Degree of Velocity acquir'd in the first Part of Time: And likewise BD (=AB) to represent the second equal Part of Time, and FE (=BC =DF) the second Degree of Velocity, acquir'd in the second Part of Time, and equal in it self to that Degree of Velocity acquir'd in the first Part of Time: and consequently AD to represent two equal Parts of Time, and DE to represent two equal Degrees of Velocity. Wherefore the Triangle ABC will represent the Space gone thro' in AB, the first equal Part of Time, with the Velocity BC; and the Triangle ADE will represent the Space gone thro' in AD two such equal Parts of Time, with the Velocity DE (the Double of BC.) But the said similar Triangles ABC and ADE are (by *Theor. 10* of *Young Gentleman's*

tleman's Geometry) as the Squares of their homologous Sides AB and AD; that is, the Spaces gone through in the Times AB and AD are as the Squares of the said Times. Thus (*Fig. 35*) ADE is actually divided into four Parts, each equal to ABC.

4. *The Spaces describ'd by the Fall of heavy Bodies, consider'd in each equal Part of Time distinctly by it self.*

Hence it is obvious, that the Spaces describ'd or gone through by a Body in its Fall, in each (equal Part or) Minute of Time, are according to the Series of the natural Numbers, 1, 3, 5, 7, 9, &c; these being the Differences of the Squares 1, 4, 9, 16, &c. Thus if a Body falls one Mile in the first Minute, and four Miles in the second Minute, it will therefore, during the second Minute, consider'd distinctly from the first, fall three Miles. And in like manner, if a Body falls at the End of two Minutes four Miles, and at the End of three Minutes nine Miles, it will, during the third Minute consider'd by it self, fall five Miles, and so five Times as far as it did the first Minute.

5. *How to find, what Space a Body will go thro' in a given Time.*

Hence it is obvious, that the Space gone through in a determinate Time by any Body being known, thereby may be found out, what Space it will go through in a given Time; viz. by finding a fourth Proportional. For Example, suppose a Body to fall twenty-four Foot

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Foot in one Minute, and it be requir'd to tell, How far the same Body will fall in the same Medium in Three Minutes; the Answer will be, 216 Foot. For as 1 to 9 (*i. e.* as the Square of one Minute is to the Square of three Minutes) so is 24 (the Space gone through in the first Minute) to 216, which therefore is the Space the Body will go through in three Minutes.

In like Manner, the Time being known, wherein an heavy Body descends through a determinate Space, it is obvious to find, in how long Time it will descend thro' another given Space, *viz.* by finding a fourth Proportional. For Example, suppose a Body has taken up one Minute in falling twenty-four Foot, and it be requir'd to tell, what Time it will take up in falling 216 Foot in the same Medium; the Answer will be three Minutes. For as 24 to 216 (*i. e.* as the first Space given is to the second Space given) so is 1 the Square of the given Time to 9 the Square of the Time requir'd, which therefore is three Minutes.

As a Body in falling acquires equal Degrees of Velocity in equal Times, so on the contrary, in rising it loses equal Degrees of Velocity in equal Times. For the Gravity of the Body thrown up continually drawing it downwards, its Motion

6.

To find, what Time a Body will take up in descending thro' a given Space.

7.

Of the Ascend of Bodies.

on upwards will continually decrease, in proportion to the Force downwards produc'd in equal Times by the Gravity of the Body. But it has been shewn afore (§. 2.) that the Motion downwards is uniformly accelerated; and therefore the Motion upwards must be *uniformly retarded*. Whence it follows, that such a Body goes through the same Spaces in equal Times, in rising as it does in falling, but in an inverted Order. Namely, if the Body be five Seconds in rising to the Height of twenty-five Foot, and the Space ascended through the first Second of Time be nine Foot, the said Body during the second Second of Time, will ascend through a Space of seven Foot; and in the third Second a Space of five, and the fourth a Space of three, and in the fifth and last a Space of one Foot. Then it will be as it were in *Æquilibrio*, *i. e.* the Impetus upwards being now just equal to the Impetus downwards, the Body will neither rise nor fall for some Time. After which it will begin to fall in the like inverse Proportion, *viz.* so as that the first Second it will fall a Foot, the second three Foot, the third five, the fourth seven, and the fifth and last nine Foot, thus taking up five Seconds of Time to descend twenty-five Foot, as it took up the same Time to ascend twenty-five Foot.

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By the same Principles it comes to pass, that the Force which a Body acquires in falling, will make it rise up again to the same Height which it fell from. For Instance, suppose (Fig. 36) the (†) Pendulum, i. e. the Body or Weight fasten'd to one End A of the String AB, whose other End is fasten'd to the fix'd Point C, as the Center of its Motion: Suppose, I say, the Body to fall from D to A, it will rise again to E, which is of an equal Height with D. For divide DA into any Number of equal Parts, suppose three, markt *a*, *b*, *c*, and EF into as many markt *d*, *e*, *f*, E. By the foregoing Principles it is obvious, that whatever Force or Degree of Velocity the Body acquires by its Gravity in descending from D to *a*, it will acquire twice as much in descending from *a* to *b*, and thrice as much in descending from *b* to *c*; with which triple Force it will begin to ascend from *c* towards *d*. But in ascending from *c* to *d* the Body will by its Gravity drawing downwards lose one of the three acquir'd Degrees of Velocity or Force; and in ascending from *d* to *e* it will lose two of the said acquir'd De-

Why Bodies rise up again to the same Height, from which they fell.

(†) It is so call'd from the Weight's *hanging* on the Thread or Wire, to which it is fasten'd; the Latin Word *Pendulus* denoting any Thing that thus hangs.

grees, the Force of Gravity downwards at f being double to its Force at d , and so destroying two of the acquir'd Degrees of Velocity or Force, wherewith the Body at c began to ascend. And likewise at E , the Force of Gravity downwards will be triple to that at d , and so destroy all the three acquir'd Degrees of Force, wherewith the Body began at c to ascend: And therefore the Body will ascend no higher, but begin to fall downwards again towards c or A .

9.
The Resistance of the Air not consider'd in the foregoing Cases.

But now it is to be observ'd, that in this and the foregoing Cases, the Resistance of the Air, wherein the Bodies are conceiv'd to move, is not consider'd; which is the Cause why the foregoing Theorems will not exactly agree with Experiments; and particularly as to the Pendulum, which, by means of the Resistance of the Air, does not return exactly to the same Height from which they fell, and consequently have not a perpetual Vibration or Motion forwards and backwards, but stand still in some Time.

10.
Pendulums ascend and descend thro' equal Spaces in equal Times, &c.

Abstracting from or setting aside this Resistance of the Air, it follows from the foremention'd Principles, that the Pendulum does ascend and descend thro' the equal Spaces DA and AE in equal Times. To which I shall here add as worth remarking, that the Longitudes
of

of Pendulums are as the Squares of the Times, wherein they perform their Vibrations.

As Gravity is that natural Principle which causes Bodies to descend, so no Body will descend, but when its Center of Gravity can descend. Thus the inclin'd Body ABDE, (*Fig. 37.*) which is set on the Horizontal Plane HP, cannot fall toward the Part D, to which it inclines, because by such a Change of Situation its Center of Gravity C would rise; as may be known by describing from A (as the Center) the Arch CF, which is the same, as C the Center of Gravity would make about the Point A, if the Body ABDE could fall; for it is evident that one Part of that Arch rises above the Point C, and consequently the Center of Gravity in such a Case must for some Time ascend of its own Accord, which is contrary to the Nature of heavy Bodies.

II.
How the Fall of Bodies depends on their Center of Gravity.

But on the contrary, the inclin'd Body ABDE (*Fig. 38*) must of necessity fall towards the Part D, which it inclines to, because its Center of Gravity C may descend in such a Situation; as will appear by drawing as afore) from A through C the Arch CF, which is the same that will be describ'd by C the Center of Gravity about A, when the Body ABED falls :)
(D) For

II.
The same further illustrated.

For all the Points of the said Arch fall below the Point C.

13.
*Of Bodies
standing
firmly.*

Hence it appears, that to have a Body keep firm upon any Thing, that supports it, and is not it self inclin'd, the right Line CF drawn from the Center of Gravity in the said Body toward the Center of the Earth, and wherein the said Body endeavours to descend (*i. e.* in short the Line of Direction) must necessarily fall in some Part of the Base or Foot AB of the said Body ; as *Fig. 37.* Otherwise, if the Line of Direction CG falls without the Basis, the Body will necessarily fall down, as in *Fig. 38.*

14.
*Further of
the same.*

Whence it follows, that by how much less the Basis of a Body is, even though it be not inclin'd, so much more easily will it be mov'd out of its Place ; because the lesser Change of Position is capable to make the Line of Direction be without its Basis or Foot. Which is the Reason that a Bowl rolls easily upon a Plane, and that a Needle will not stand upon its Point. It is obvious, that agreeably to what has been said, it follows also, that the wider the Foot of a Body is, the more firmly will it stand, because so much the greater Change is necessary to cause the Line of Direction to fall without the Foot of the said Body.

It

It appears also from what has been said, that if the Plane OP, which sustains the Body ABDE (*Fig. 39*) be inclin'd, that Body will slide, when its Line of Direction CG falls upon any Part of its Base AD. And that a Body *abde* will fall, when its Line of Direction *cg* falls without the said Base *ad*.

15.
Of the Sliding and Falling of Bodies.

Whence it follows, that a Bowl set upon an inclining Plane, as the Roof of an House, will roll off it, because its Line of Direction not being perpendicular to the Plane, (but to the Horizon,) cannot pass through its Foot where it touches the Plane, it being almost an indivisible Point.

16.
Further of the same.

It is remarkable, that we naturally observe this Law of Statics, we are here speaking of, to keep our selves from falling. Thus, when we would rise from a Seat, we bend our Body, by thrusting forward our Head and Back HB, and putting backwards our Feet and Legs FK (*Fig. 40*) so as the Line of Direction LG of our Center of Gravity may pass through our Footing F or Basis. And it is obvious, that if we bend not enough, we fall backwards or towards B; but if we bend too much, we fall forwards or towards F or K.

17.
Why a Man bends, when he rises from a Seat.

18. In like Manner, when we go up Stairs, as we lift one Foot up, so we bend our Body forward, that we may (both go up easier, and also) may preserve our selves from falling backwards, by having the Line of Direction of our Center of Gravity too far from our Footing, which in this Case is chiefly the Foot lifted up upon the Stair, when we are getting up.

19. On the same Account it is, that when a Man carries a Burthen on his Back, he leans forward; but if he carries a Burthen before him, he leans backward. If he carries it with his right Hand, he leans to his Left; if with his left Hand, he leans to his Right. Namely, that hereby he (may not only more easily carry the Burden, but also) may preserve himself from falling that Way which the Burden he carries, pulls him.

20. And that we learn this, not from Reason, but rather from natural Instinct, is evident, not only from Mens doing so without knowing the Reason of it, but also from irrational Creatures doing the like. For a Goose going through a Barn or other Door, be it never so high, does thrust forth, and so downwards his Head, not because he is afraid of striking it

Why a Man leans forward when he goes up Stairs.

Other common Postures accounted for.

The like Rules observ'd by irrational Creatures.

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it against what is so much above it, but) in conformity to this Law of Staticks. Namely, at a Barn Door there is usually a Threshold or the like, to be gone over. Wherefore the Goose having put one Foot on the Threshold, thrusts forward his Head, that his Center of Gravity may not be too far behind his Feet, and so pull him backwards; but that by thrusting his Head and Neck so far as is requisite beyond the Threshold, the Direction of his Center of Gravity may be brought so near to his Footing, as that he may go over the Threshold more easily, and without falling either backward or forward. So that it appears, the Goose is not to be laugh'd at for so doing, but rather they, who neither know the Reason of his so doing, nor at least that he does no other than they do themselves in like Cases. And this is sufficient to our Design, to have taken notice of concerning Staticks, so call'd in a more restrain'd or proper Sense.

C H A P. IX.

Of Hydrostaticks.

Hydrostatics, what.

TH O' the Word *Statics*, both as to its own literal Import, and also in its largest Acceptation, denotes the Science of the Weights of Bodies in general, whether fluid (or liquid) or not fluid, and whether weigh'd in the Air or Water; yet it is usual to restrain *Statics* in a more appropriated Sense to denote the Science of the Weights only of solid (or not fluid) Bodies, and that too when weigh'd in the Air; and to give the peculiar Name of (*) *Hydrostatics* to that Part of *Statics* largely taken, which treats of the Weight of fluid or liquid Bodies, especially of Water, and of solid Bodies put into (†) liquid Bodies, especially into Water.

The

(*) It is a Greek Word, compounded of *στασις* the Science of Weights, and *ὕδωρ*, Water.

(†) There is this Difference between a *fluid* and a *liquid* Body. A Fluid is a Body, whose Parts are easily separated, and being separated join together immediately, as

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The primary Property of heavy Fluids are these; that if an heavy Fluid ABCD (*Fig. 41*) be, either not press'd at all, or equally press'd from above; its upper Surface AB will lie horizontal or level. And if the said Fluid be disturb'd, it will by its own Gravity return to the same Level of its upper Surface. For the lower Particles of the Fluid will not, by Reason of their Gravity, ascend upward of themselves, to raise any Part of the said Surface above the other; nor can the upper Particles of the Fluid descend, since there is no Room for them, in the lower Part of the Vessel, it being taken up by other Particles of equal Weight. And therefore if there be no Pressure at all from above, or an equal Pressure, then there is nothing to disturb or alter the Level of the upper Surface. But if it be disturb'd, the fluid will return to its Level, because the higher Part HI (*Fig. 41*) being heavier, will partly depress the Particles under it, and will partly run down into the lower Part;

Air, Flame, Water, Oil, and other Liquors. A Liquid is a Body whose Parts are also easily separated, and again come together immediately, and withal will continually flow or spread themselves, till their upper Surface becomes level, as Water, Oil, but not Flame. Hence it appears, that all Liquids are Fluids, but all Fluids are not Liquids.

(D 4)

and

and that so long, as any one Part of the Surface is higher than the others.

3. *Further of the same.* If the Fluid (*Fig. 42*) be from above unequally press'd, it is obvious, that the Part thereof AE, which is most press'd, will descend, the Particles so press'd, thrusting out of their Places the other Particles, that are either not press'd at all, or less press'd; which therefore will ascend to HI in proportion to the Descent FG of the Particles press'd AE. Namely, as the Part press'd AE is equal to the Part not or less press'd EB, so the Ascent EH or BI will be equal to the Descent AF or EG.

What has been said of the upper Surface AB, holds good of any other Parallel Surface within the Fluid, as EF *Fig. 43*. Namely, if it be equally press'd by the upper Part of the Fluid, and withal by any Thing else swimming either on the Top or within the Fluid, it will retain its horizontal Situation or Level. But if it be any where press'd more than in other Parts of it, there it will sink, the Parts of it which are less press'd ascending as that descends. And from what has been hitherto said, are inferr'd the following Propositions, as so many Corollaries.

If an heavy Body EFHB (*Fig. 44.*)
lying upon a Fluid AB, be of an equal
Weight with the Air that takes up an e-
qual Space AHEG, the Surface AB of the
Fluid will retain its Level, as being eve-
ry where equally press'd.

4.

Corollary
the first.

If the Body EFIK (*Fig. 45*) be lighter
than the Air which takes up the like
Space, then the said Body, and the Par-
ticles of the Fluid under it, will ascend,
'till the Aggregate (of the Body IKEF and
of the Parts EFHB of the Fluid so as-
cending, *viz.*) IHBK be of equal Weight
with the Air, that takes up the like Space
GAHI. For then the Surface AB will be
equally press'd every where, both in AH
and HB.

5.

Corollary
the second.

If the Body EFAB, (*Fig. 46.*) be hea-
vier than the Air of a like Bulk, then
the Body will swim in the Fluid, de-
scending so far into it, 'till the Aggre-
gate (of the Air GEIK and the Fluid
IKAH, *viz.*) AHEG, which take up to-
gether like Space with the said Body, be
of equal Weight with the Body. For
the Surface AB will become equally
press'd, both in AH and HB.

6.

Corollary
the third.

If the Body EGDF (*Fig. 47*) be hea-
vier (not only than the Air, but also)
than the Water (or other Fluid) that
takes up a like Space HCGE, then the
Body will sink down quite to the Bottom

7.

Corollary
the fourth.

CD.

CD, there being nothing to counterpoise or bear it up, 'till it comes to the Bottom. For the Body being suppos'd to be put on the upper Surface AB, and to lie upon the Part IB thereof, it being heavier than a like Bulk of Air, the Part of the Fluid IBDE will be more press'd thereby, than the other Part of the Fluid AIEC by the Air, and therefore the Particles in IBDE will descend. And in like Manner when the Body is come to GF, it being heavier also than a like Bulk AHGI of Water, it will still press the Parts of the fluid Water GF more than are those that are under GH, and therefore will cause the Parts under GF still to give way, 'till it comes to the Bottom CD.

8.
*Corollary
the fifth.*

Upon the same Principles, if a Body GHIK (*Fig. 48*) be immerg'd into the Fluid, and be (of equal Weight with so much EFGH of the Fluid as takes up the like Space, *i. e.* in short, be) of the same specifick Gravity with the Fluid, then the said Body will stay where it is plac'd in the Fluid, without either rising or sinking. For the Surface EI being equally press'd all along, the said Body cannot rise. And the Surface FK being all along equally press'd, neither can the said Body sink.

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But if the said Body be specifically lighter than the Fluid, and consequently the Bulk GHIK be lighter than the Bulk EFGH, it evidently follows, that the Surface FK will be more press'd in FH than in HK, and consequently the Body GHIK will be thrust up and rise still higher and higher for the same Reason, 'till it comes to such a Situation, as that it is of equal Gravity with so much Air (which is in this Case suppos'd specifically lighter than the Body) and so much of the Fluid, as take up equal Space with it ; (as is represented *Fig. 46*) according to §. 6. But lastly, if the Body be of the same specifick Gravity with the Air, then it will rise just to the very Top of the Fluid ; as is represented *Fig. 44.* according to §. 4.

From what has been said, it is easie to account for several Occurrences in Fluids, as particularly in Water ; of which the more remarkable or common shall be taken Notice of. Hence then it is easie to tell, why we don't feel the Weight of the Bucket and the Water in it, when we draw Water, 'till it begins to rise out of the Water. Namely, because supposing the Bucket with the Water in it to be represented, *Fig. 48*, by GHIK, it being of the same

† Gra-

9.

Why a Bucket of Water has little or no Weight under Water.

The Young Gentleman's

(†) Gravity with the like Bulk of Water EFGH, the Surface FK will be equally press'd, both in FH, and HK ; and therefore the Bucket of Water can't descend, but is held up by the Water under it ; and consequently the Weight of it is no more felt, than is a Weight which is counterpois'd by an equal Weight upon a Balance. And the same holds all the while the Bucket is drawing up to the Surface AB. Where it, and the Water in it, coming into the Air so much specifically lighter than the Water or Wood, &c. of the Bucket, the Air can't sustain it, and so the Weight of it must be sustain'd then by him that draws up the Bucket.

10.
Why the
Depths of
some Seas
cannot be
fathom'd.

On the same Principles the Reason, why one cannot fathom the Depth of the Sea in some Places, is this : Because tho' the Plummets or Lead made use of be specifically heavier than Water, yet the Line to which the Plummets is fasten'd, is (in this Case) specifically lighter than the

(†) The Water in the Bucket is certainly of the same specifick Gravity with the Rest of the Water. And the Bucket, if all of Wood, is much lighter. And if otherwise, the specifick Lightness of the Wood does reduce the specifick Gravity of the Iron about the Bucket, almost to an *Æquilibrium* with the Water,

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Water. Wherefore when the Depth is so great, that such a Quantity of Line is requir'd, as that the Weight of the Line and Lead together is no greater than the Weight of a like Bulk of Water, then the Lead or Plummet will sink no deeper; and consequently will not shew the Depth of the Sea in that Place.

According to the same Principles we may easily tell, why one Liquor will swim upon another, or remain at Top without mixing, if it be pour'd on gently; as Oil in respect of Water; and Water in respect of Quicksilver. • Namely, because Oil is specifically lighter than Water, and Water than Quicksilver.

II.
*Why some
Liquors
swim upon
others.*

Hence likewise it ceases to be a Wonder, that a Ship, which has sail'd in the open Seas, has sunk and been lost in the Mouth of a River of fresh Water; namely, because Sea-water is much heavier than fresh Water. And therefore though the Ship with its Burden was lighter than a like Bulk of Salt-water, and so was sustain'd by the Salt-water in the Sea, yet it was heavier than a like Bulk of fresh Water, and therefore sunk in the River. It is obvious, that for the same Reason any Ship or Boat will draw more Water (that is, though it does not sink quite down to the Bottom, yet will sink deeper) in fresh Water than in Salt-water.

12.
*Why Ships,
that sail in
the Ocean,
sink in
fresh Water.*

More-

13.
*Why Wood,
 that will
 swim for
 some time,
 sinks after-
 wards.*

Moreover the Reason, why a Log of Wood, that has been swimming a long time upon Water, will sink at last, is this, *viz.* because such a Piece of Wood may be of the same or greater specifick Gravity than Water, setting asides the Pores of the Wood; but by reason of its Pores, the Air which fills the said Pores and the Wood together, make up one Whole lighter than an equal Bulk of Water, and therefore the Wood will not sink at first. But after some Time the Water being got into the Pores, and so the Air excluded, then the Water in the Pores and the Wood make up one Whole heavier than the Water of a like Bulk, and so must sink according to the Principles of Hydrostaticks aforementioned.

14.
*Why hollow
 Vessels of
 Brasse or Iron
 will swim, when
 Lumps of
 the same
 Metal will
 sink.*

From which Principles it follows also, that although a Lump of Brasse or Iron or other Metal will sink in Water, as being specifically heavier, yet a Kettle or the like of Brasse or Iron (*&c.*) will not sink but swim; because the Kettle with the Air contain'd in its Hollow, make up one Whole, which is much lighter than an equal Bulk of Water.

15.
*A Body will
 weigh less
 in Water,
 than in Air.*

It follows also, that a Body will weigh less in Water than in Air, by just so much as is the Weight of an equal Bulk of Water. Whence it comes to pass, that two Pieces of Metal of different specifick

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Gravities, suppose one of Gold and the other of Silver, which shall be of equal Weight in the Air, will not be so in the Water, that Metal whose specifick Gravity is greatest, (*viz.* the Gold) losing less of its Weight in the Water than the other (*viz.* the Silver,) because it takes up the Room of a less Bulk of Water.

If (*Fig. 49*) a Fluid, particularly (Mercury or) Quicksilver contain'd in the Tube CD presses on the Part C of the Surface AB, more than the Air presses on the other Parts, C will descend, and the Quicksilver run down out of the Tube, 'till what is left CI in the Tube presses C, just as much as the other Parts are press'd by the Air. And then no more will run out of the Tube, because of the equal Pressure in all Parts of the Surface AB. Where it is to be noted, that the Weight of the Quicksilver CI is equivalent to the Weight of a Column of Air, extended from the Surface AB up to the Top of the Atmosphere, and of the same Basis or Diameter of the Tube CD.

16.
Why a Fluid in a Tube will stand up higher than the same Fluid without the Tube.

And it is observable (*Fig. 50*) that the Quicksilver will rise up just to the same Height in the oblique or sloping Tube EG, as it does in the upright Tube DC; although the Quick-silver EF in the Tube EG be more, and consequently heavier,

17.
Why a Fluid will rise to the same Height in a sloping Tube, as in an upright

heavier, than the Quicksilver CI in the Tube CD. The Reason whereof is this, that as much as the Quicksilver EF is more than the Quicksilver CI, so much greater is the Basis E than the Basis C, (this being equal to the Basis of the Cylinder EK, wherein the Quicksilver EH will be found to be just so much as the Quicksilver EF, by reason of the two Tubes EK and. EG being Cylinders of Equal Height upon the same Bases, and therefore Equal one to the other.) Now it is obvious that supposing the Basis E to be Twice as big as the Basis C, and consequently the Quicksilver EH in the Tube EK (or the Quicksilver EF in the Tube EG) to be Twice as much as the Quicksilver CI in the Tube CD, it evidently follows that as the Quicksilver CI do's equiponderate to a Column of Air of the same Basis C, so the Quicksilver EH (or EG.) which is the Double Quantity of CI, do's but equiponderate to a Double Quantity of Air, *i. e.* to a Column of Air, whose Basis E is the Double of the Basis C.

18.
*Further of
the same.*

Since according to the Rules of Statics, Bodies gravitate or weigh heavier in a Perpendicular, than in any Oblique or sloping Situation, it may be ask'd, why the Quicksilver EH gravitating or pressing upon the Basis E more,
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than do's the Quicksilver EF, (that being perpendicular to E, this oblique.) the Quicksilver do's not descend lower in the Tube EK, and ascend higher in the Tube EG. The Reason then is this, that as much as EH presses downwards more than EF, so much do's the Force at E, which (arises from the Pressure of the Air on the other Parts of the Surface AB, and) sustains the Quicksilver in each Tube, press upwards more in the upright, than in the oblique Tube; and both for the same Reason, *viz.* because the Pressure, whether upward or downward, is in the Tube EK perpendicular, whereas in the Tube EG it is oblique, and therefore not so great. Now the Force at E, which holds up the Quicksilver in the upright Tube, being as to Effort so much greater than the Force at E which holds up the Quicksilver in the oblique, as the Weight of the Quicksilver in the upright Tube, is greater than the Weight of the other; it follows, that the said Force at E acting upwards according to the Perpendicular EH, is as able to sustain the Quicksilver in the upright Tube, as is the same Force acting upwards according to the oblique Line EF to sustain the same Quantity of Quicksilver in the oblique Tube. And therefore the Quicksilver in

(E) the

the upright Tube will not descend lower by Running any of it out of the said Tube, nor will the Quicksilver in the oblique Tube ascend any higher, by any more Running into it: but the Quicksilver in both Tubes will keep at an equal Height.

19. *The Shape of the Vessel makes no Difference as to the Rising of the Fluid.*

And the same will hold as to any other Tube (or Vessel) of any other Shape: Namely, the Height of the Quicksilver will be the same in any Vessel, as it is in a Cylinder of the same Height and Basis with the Vessel. Thus, for Instance, *Fig. 51*, the Height of the Quicksilver is the same, both in the round-headed Tube *DEHF*, and the sharp-headed *KLB*, as it is in the Cylinder inscrib'd in the former, *viz.* *CI*, or the Cylinder *BLOP*, circumscrib'd about the latter. For as to the round-headed Tube *DEHF*, as much of the Quicksilver as lies without the Cylinder *CI*, is sustain'd not so much by *C*, as by the Parts *MT* and *NS* of the Tube's Head which are under them, and no more of the Quicksilver is properly or directly sustain'd by *C*, than what lies within the Cylinder *CI*. And consequently the Effect will be the same, as if the Quicksilver was put in the Cylinder *CI*.

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As to the sharp-headed Vessel LBK, and the Cylinder BLOP, the Quicksilver will likewise rise as high in the one as the other. For as to so much of the said Vessel, as allows a free Ascent from the Surface AB, the Quicksilver will there rise to its just Height CI, for the same Reason as it will rise to the same Height in the Cylinder BLOP. As to the other Parts of the said Vessel, where the Quicksilver is hinder'd by the Sides of the Vessel to rise up to the Height CI, as much as is there wanting in the Weight of the Quick-silver, which is over the respective subjacent Parts of the Surface AB, so much is supply'd by the Sides of the Vessel. For although the Sides of the Vessel do not press downwards as a contrary Force, yet they keep the Quicksilver from being thrust Higher by the Force that presses upwards. And consequently the several Parts of the Sides may be look'd on as so many Impediments, which are equivalent to the Weight of so much Quicksilver, as would be contain'd between the respective Parts of the Sides, and the Parallel to I. And so the Surface AB will be equally prest all along under the Vessel B.

21.
Why the
Fluid rises
higher in a
Tube at the
Bottom,
than at the
Top of a
Mountain.

The Pressure of the Air being that which makes the Quicksilver rise in the Tube; it follows, that the greater is the Pressure of the Air, the higher will the Quicksilver rise; which is the Reason, that the Quicksilver will rise higher in the same Tube at the Bottom of a Mountain or considerable Hill, than at the Top. Namely, because the Column of Air pressing the Surface AB will be so much longer at the Bottom, than at the Top of the Mountain, as is the Height of the Mountain; and consequently the said Column containing so much more Air, will press so much more.

22.
Of the
Weather-
Glasses.

So as to the (*) *Barometer* or *Weather-Glass*, the Quicksilver rises and falls in the Tube, as the Air is heavier or lighter, and so more or less presses the Quicksilver that is in the Well (as it is call'd) of the Barometer. And thus far is agreed by all: but what it is that causes the Air to be sometimes heavier, sometimes lighter, is not so agreed. That the Vapours do not, (as is, or has been, commonly suppos'd,) increase the Gravity of the Air, seems more than

(*) It is so call'd as being a *Measure of the Weight* of the Air, the Word *βάρος* signifying *Weight*, and *μέτρον* a *Measure*, in the *Greek* Tongue.

probable

probable on this Account: Namely, that when the Vapours are in the lower Region of the Air, as in rainy Weather, than the Air is lightest, as appears by the Falling of the Mercury (or Quick-silver) in the Barometer. And when the Vapours are in the middle Region of the Air, (*i. e.* when the Clouds are high,) they do not increase the Pressure of the Air. For although the Air be heavier then, it is not because the Clouds are high; but the Clouds are high, because the Air is heavy. For when the Air near the Earth is more dense than usual, then it becomes heavier than the Vapours, which therefore must ascend, and at last settle in that Region of the Air, which is of the same Gravity with themselves. If the Vapours were the Cause of Increasing the Air's Gravity, there must be as many Vapours in the Air at a Time, as are equal in Weight to three Inches of Mercury, for so much we find the Mercury rises or falls in the Weather-Glass. Now Mercury is about 14 Times heavier than Water, and consequently there must be in the Air at once so many Vapours, as are equal in Weight to a Column of Water of 42 Inches in Height, and whose Basis is equal to the Surface of the Earth; which is much more than falls down in Rain, during a whole Year.

(E 3)

For

For a whole Year's Rain do's not fill a Vessel above 14, or 15 Inches high, as is observ'd in the *History of the Royal Society at Paris*.

23.

The probable Cause of the Rising and Falling of the Mercury in a Weather-Glass.

The Reason then, why the Air is heavier at one Time than another, arises more probably from there being more Air on that Part of the Earth's Surface, when the Air grows heavier. And this proceeds from Winds. For Example; If the Wind, which is nothing but a Stream of Air, should blow on any Place, and the Air thus mov'd should be kept in that Place by Mountains or Hills: or, if two contrary Winds should blow on the same Place; in either Case the Air will be heaped up in the Middle; and consequently there being more Air its Gravity will be increas'd. But if the aforesaid Circumstances, or the like do not happen in any particular Country, then the Air which is over it, will grow less in Quantity, and consequently lighter. Whence it is probable, that the Winds are the Causes of the Variation of the Air's Gravity.

24.

Of the Siphon, or Instrument us'd to draw off Wine or other Liquors out of Vessels.

It was afore (§. 16.) intimated, that what has been said in that and the following Paragraphs concerning Mercury or Quicksilver, holds true likewise in respect of other Fluids. But the fore-mention'd Affections have been illustrat-

ted,

ted, particularly as to Quicksilver, forasmuch as it is that which is us'd in Weather-Glasses, wherein the said Affections are more frequently seen by Us: It being the Design of this Treatise by the Laws or Principles of Mechanicks to explain chiefly such Things as more frequently occur in common Use. On which Account I shall add here the Explication of the Effect of the Siphon, (or Instrument us'd by Vintners, &c. in drawing off Wine, or the like,) according to the forementioned Principles of (*Mechanicks*, or more particularly of) *Hydrostaticks*. It is then obvious, that (*Fig. 52.*) the Air being suck'd out of the Siphon at E, the Wine (or other Liquor) in the Vessel ABFG will be thrust up by the Pressure of the Air upon the Surface AB, into the other End H of the Siphon apply'd to the said Surface. And the Wine will be thrust upwards in the Siphon, 'till the Weight thereof comes to an Equilibrium with the Pressure of the external Air, suppose 'till it comes to the Height CI. If therefore the Top D of the Siphon be not higher than I, the Wine will rise to the said Top of it; and there finding a Vent or Passage down the other Leg DE of the Siphon, it will go that way, and so run out at E; continuing so to do, the Causes still continuing,

(E 4)

ning, 'till the Vessel is emptied. But it is to be observ'd, that the Mouth E of the Siphon must be lower than the Mouth H. For else, E and H being equally press'd by the Atmosphere, if the Leg DE be but of an equal Length with DH, and consequently the Fluid in one Leg of equal Gravity, with the Fluid in the other; the Fluid would stay in the Siphon, not moving one Way or the other, without some other external Force beside the Air. And by Parity of Reason, if DE be shorter than DH, and so the Fluid contain'd in DE of less Weight than that in DH, the Fluid would run (not from H to E, but) from E to H, the Air coming up at E, and thrusting the Fluid forward the contrary Way. This would happen, except at least the Siphon be so wide, as that the Air may ascend by the Sides of the Fluid, while the Fluid it self descends in DE.

25.
The Mechanism of
a common
Pump, ex-
plain'd.

Proceed we next to explain the Mechanism of a Common Pump (Fig. 53.) by the Principles of *Hydrostaticks*. The Handle KL being thrust down, the Sucker GF is drawn upwards from E, and together with it the Air that lies upon it. Wherefore the Water at the Bottom, being without the Hollow of the Pump at A and B press'd by the Air, but within the Hollow of the Pump at C being not

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(at least, not so much) press'd by the Air, will therefore ascend from C to D, and thrusting open the Valve E, (as having no Weight or Pressure upon it,) will ascend still higher to the Bottom of the Sucker, as meeting with no Pressure from above, and being continually press'd upwards from Beneath. The Water having thus fill'd the hollow Part EF of the Pump, and the Sucker FG (by lifting up the Handle KL) being forced downwards, do's force also downwards the Water under it between F and E, and by this Means forces down and keeps shut down the Valve E. Wherefore the Water between E and F being thus press'd by the Sucker; and having no Passage downwards through E, it lifts up the Valve G of the Sucker FG, this being the only Way it can make for it self. The Water thus arising above the Sucker, is together with it (the Valve G being kept down by the Weight of the Water lying upon it, so that the Water can't run back that way again; and the Handle KL being thrust downwards again) lifted upwards: where finding an open Passage, it runs out at the Mouth H of the Spout, other Water ascending up in the mean while through D, (as afore,) when the Sucker is drawn up. And thus continually, as long as the Handle is lifted

lifted up and down, and consequently the Sucker is mov'd up and down, the same Causes being thus continued.

26.

Why some
Wells will
not be made
Pump-
Wells.

It is obvious from the Principles of *Hydrostaticks* afore laid down, that the Distance between the Water at C in the Well, and the Valve E, (or Bottom of the Sucker when at the Lowest, viz. at E,) must not be greater than the Altitude, whereto the Pressure of the Air is sufficient to make the Water rise, which is found by Experience to be about 33 Foot. Hence such Wells, as have their Water lying so deep, as that (a Pump can't be made, but) the Distance will be greater than the aforesaid Altitude, can't be *Pump-Wells*, but must be *Dram-Wells*, i. e. can't have their Water drawn up by a Pump, but by a Bucket and Rope.

27.

The Tide
explain'd
by the Prin-
ciples of
*Hydrosta-
ticks*.

I shall conclude this Chapter, with shewing how that Rising and Sinking of the Sea-Water, which we call the *Tide*, (and which has been of old observ'd, but is by none of the old Philosophers, as I know of, tolerably accounted for,) may be very naturally accounted for by the Principles of *Hydrostaticks*, after such a manner as is very easy to be comprehended.

The

The Moon's Orbit being over those Parts of the terraqueous Globe, which lie between the Tropicks, and where are the largest Seas, as she passes over these Seas, she presses (those Particles of Matter which are next under Her, and they the next underneath, and so the whole intermediate Air, and thereby at last) the Water of the Sea, and so causes the said Water to rise, on each Side of that Part of the Sea which lies under her Orbit, that is, toward the North and South Parts of the World. Hence to us who live in the northern Parts of the World, the Tide always comes from the South, because the Moon's Orbit is South of us. And on the like Account, the Tides comes from the North, to those who live in the southern Parts of the World, because the Moon's Orbit is North of them. And hence it is evident also, that the Tide must come so much sooner or later to any Place, as it lies more or less distant from the Tropicks, or that Tract of the Sea which is under the Moon's Orbit.

Why the Tide always comes to Us from the South

The Tide being thus caus'd by the Pressure of the Moon, it plainly follows, that because the Moon comes every Day to the Meridian of any Place fifty Minutes later than the Day before; therefore the Tide likewise must fall out fifty Minutes

Why the Tide comes every Day fifty Minutes later than the Day afore.

Minutes later every Day, than the Day before, in Respect of any particular Place.

30.
Why the Tide is six Hours coming in, and six Hours going out.

Also, because the Moon is six Hours in coming from the Horizon of any Place to its Meridian, therefore (it is obvious, that) the Tide must be six Hours coming in. And in like manner, because the Moon is six Hours in going from the Meridian to the Horizon of any Place, therefore the Tide must be the same Time in going out. Where it is obvious, that as the Pressure of the Moon begins in respect of any particular Place, so soon as she comes to its Horizon; so her Pressure continually increases 'till she comes to the Meridian of that Place, and consequently the Tide rises still higher and higher, 'till she comes to the said Meridian. Which as the Moon leaves again, so her Pressure decreases, and consequently the Tide sinks lower and lower.

31.
Why the Tides are Bigger at the Change and Full of the Moon, than at other Parts of the Month.

Moreover it being agreeable to the Laws of Staticks, (as is found by Experiments,) for heavy Bodies to gravitate so much more, as they are nearer to the Center of the Earth; hence the Reason, why the Tides are Bigger at the New and Full Moons, than at the Quarters, is easily to be assign'd; namely, the Moon's being at her Change and Full
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nearer to the Earth, than at her Quarters.

Again it is observ'd, that as the Tides at the Change and Full of the Moon are bigger, than at any other Time of the Month; so the Tides at the Change and Full of the Moon about the Equinox's are bigger, than those at any other Time of the Year. Which may be accounted for by the Moon's being then over the Middle Part of the Ocean, which is under the Celestial Equator, and consequently by its Pressure thereon making a greater Quantity of Water recede to each Side, viz. Northward and Southward, than when she presses on one Side of the Equator, and so not in the Middle of the Ocean.

32.
Why the Tides at the Change and Full of the Moon about the Equinox's are bigger, than at other Parts of the Year.

It remains to be observ'd, that it is demonstrable (†) by the Principles of Staticks, that the Moon do's cause (the Sea to swell, or in one Word) the Tide, not only in that Part of the Sea which she is over, but also in the Part opposite thereto. And hence it comes to pass,

33.
Why there be Two Tides in every twenty-five Hours.

(†) Such as would have a more accurate Account of the Tide, let him consult *Newton's Phys. Mathem. Lib. 3. Prop. 24 & 37, and Numb. 16. of the Philosoph. Transact. Lond. 1666*; as also *Gregory's Astron. Phys. and Geom. Lib. 4. Prop. 65*. I have here contented my self with a more general Account, as being more Easy to be understood.

that

that there be two Tides every twenty five Hours; namely, because in that Time the Moon going from any Meridian, returns to it again; and consequently causes one Tide, while she is in the upper Hemisphere, and another Tide while she is in the lower Hemisphere.

34

And thus I have taken Notice of so much of *Mechanicks* (in general, or including *Mechanicks*, *Staticks*, and *Hydrostaticks* properly so called,) as is answerable to the Design of this Treatise.

F I N I S.

Fig. 1.

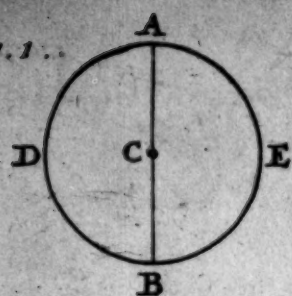


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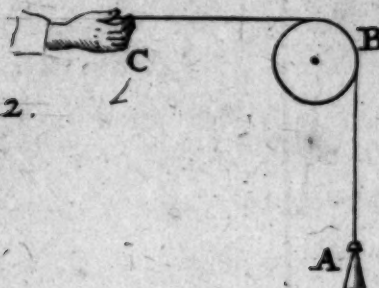


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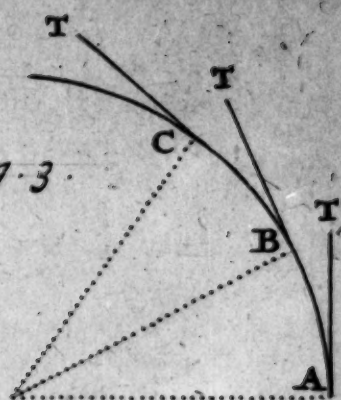


Fig. 4.

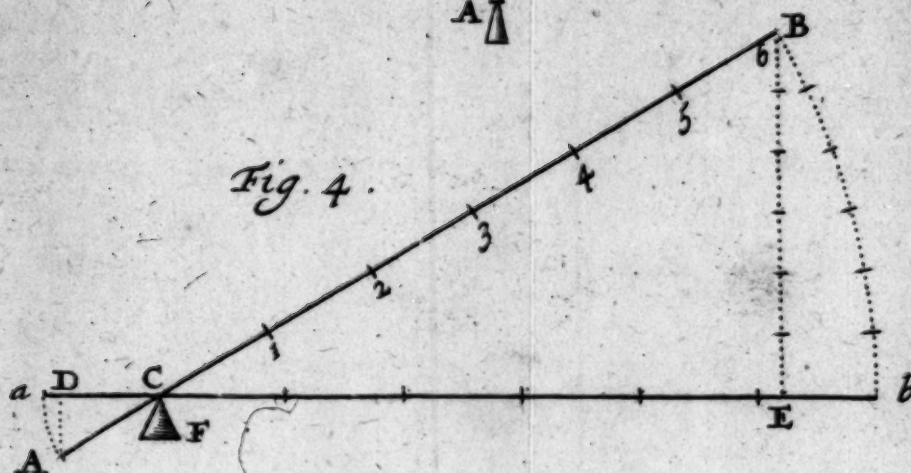
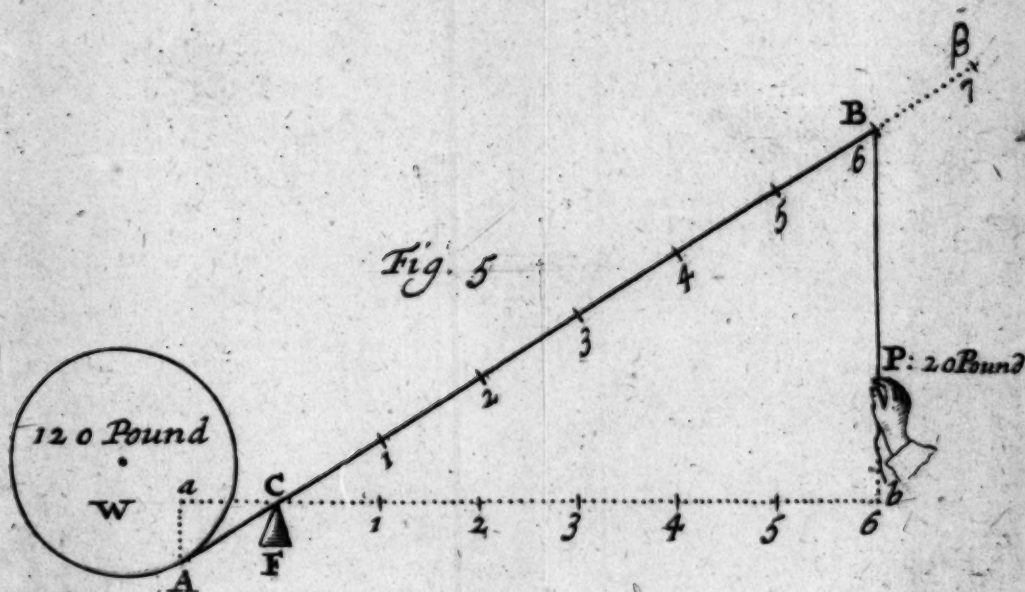


Fig. 5.



Mechan. Plate . 2 .

Fig. 6

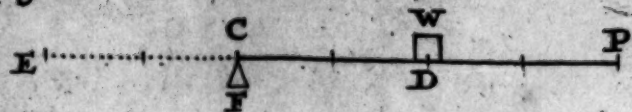


Fig. 7

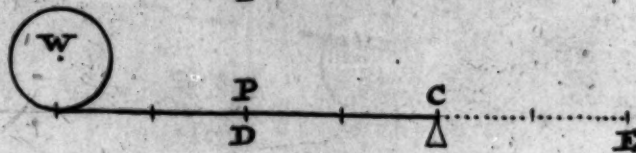


Fig. 8

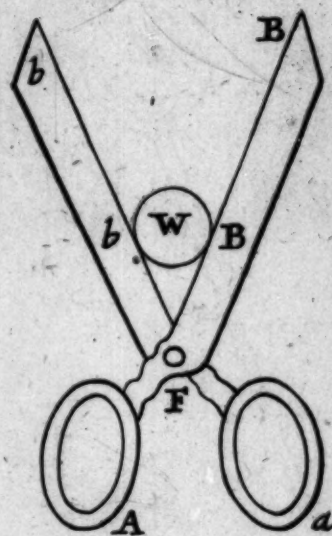


Fig. 9



Fig. 10



Fig. 11

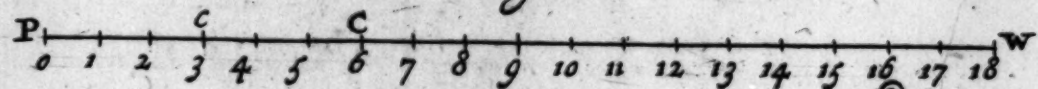


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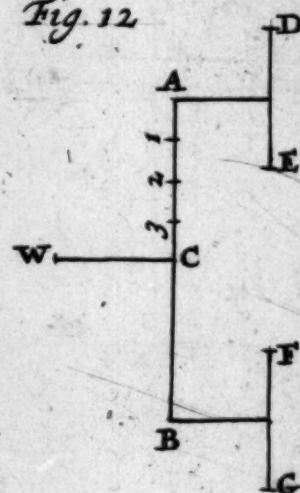
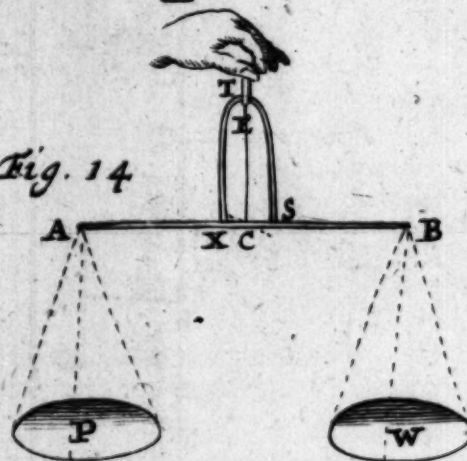


Fig. 13



Fig. 14



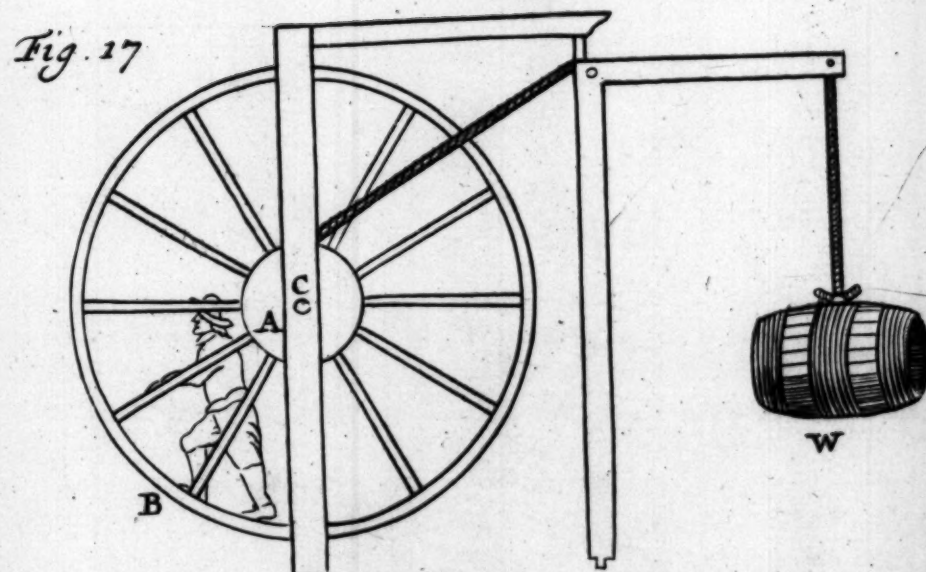
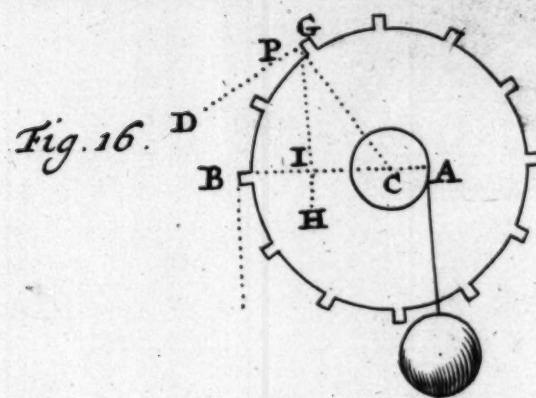
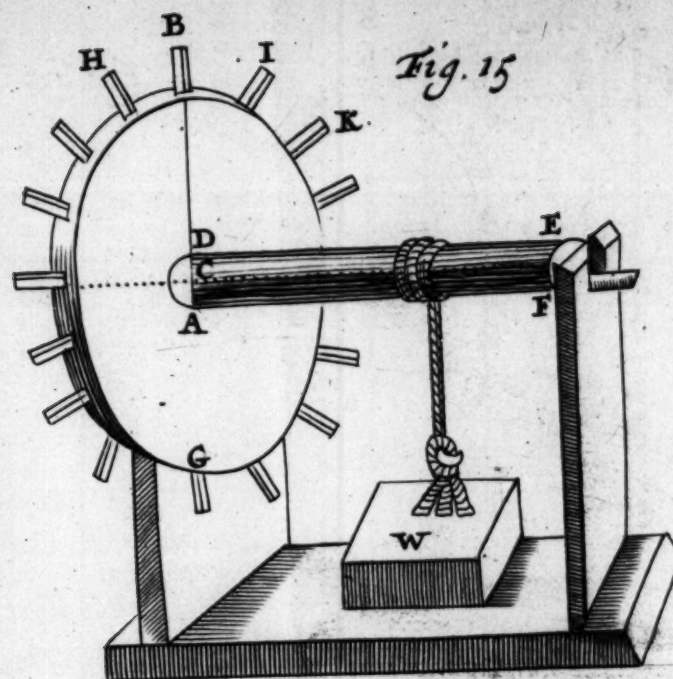


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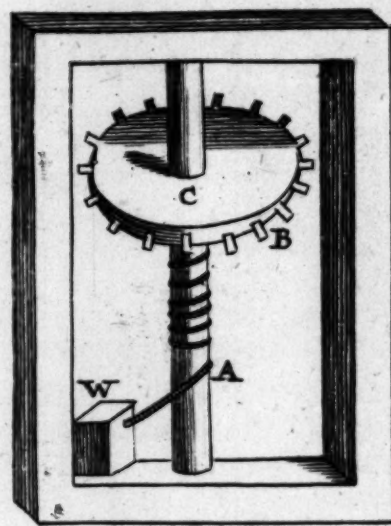
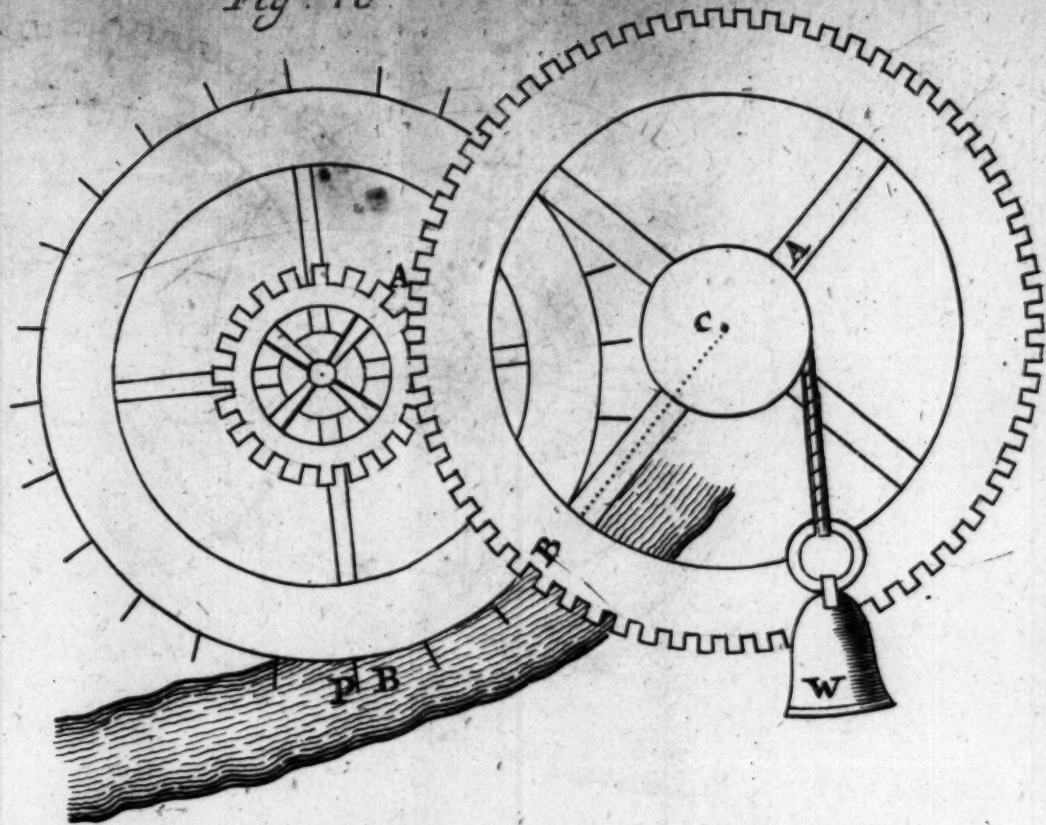


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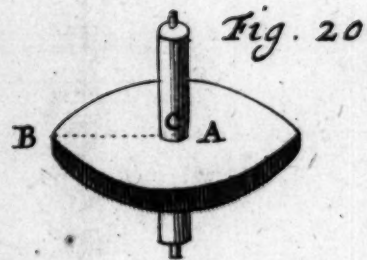


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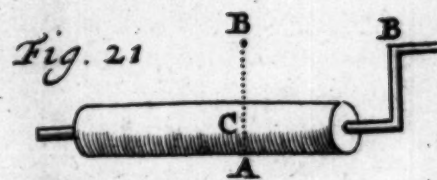


Fig. 21



Fig. 22

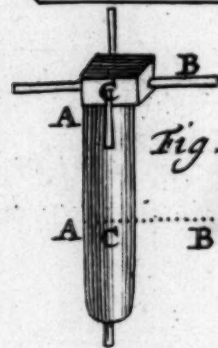


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Fig. 24

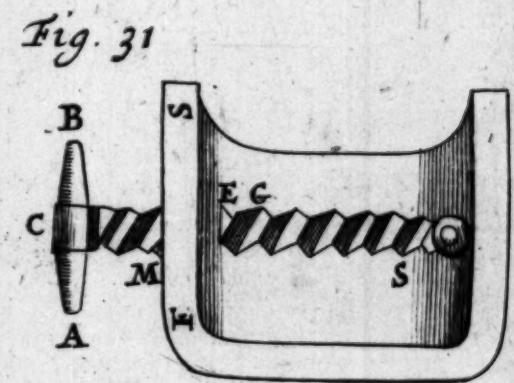
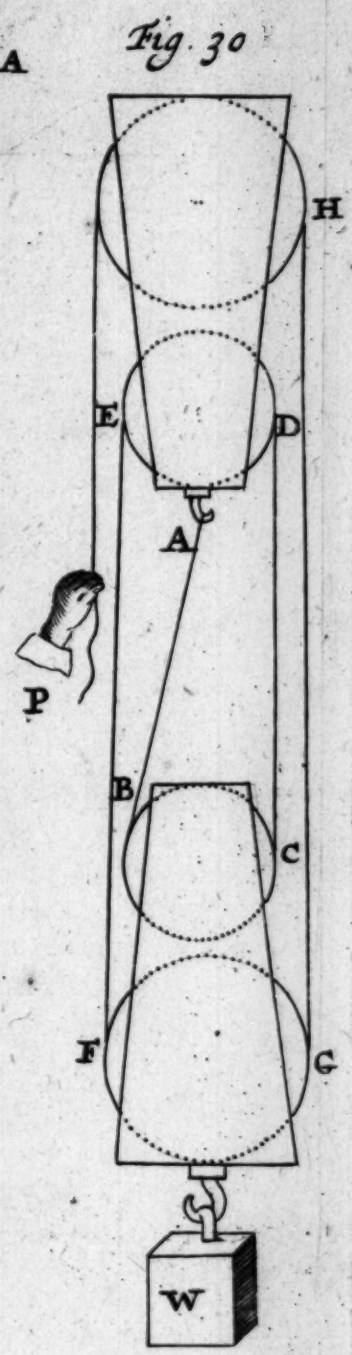
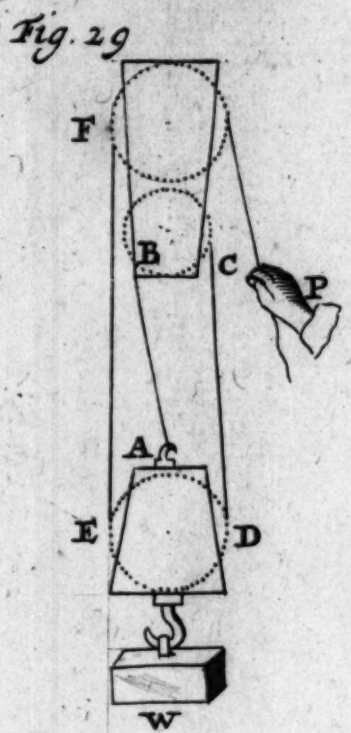
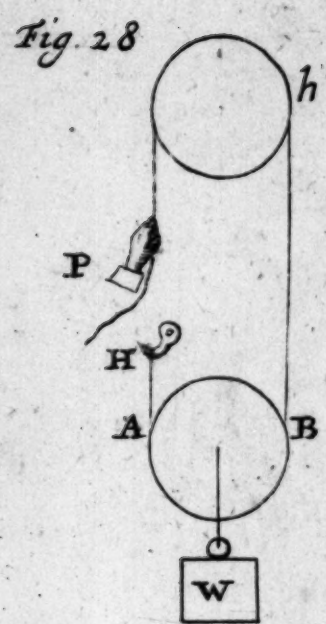
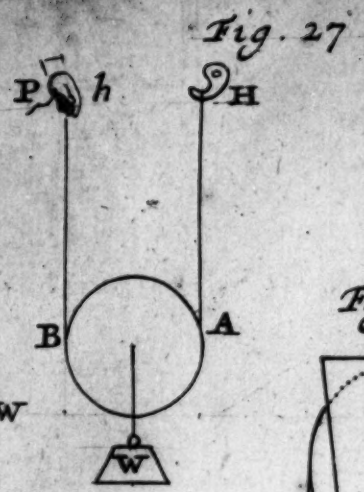
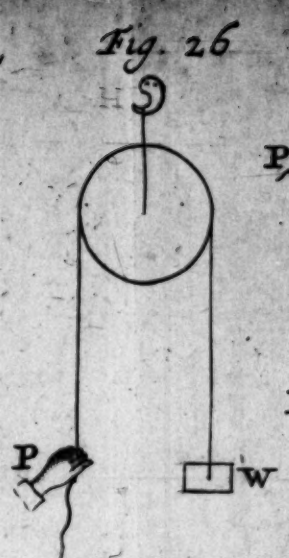
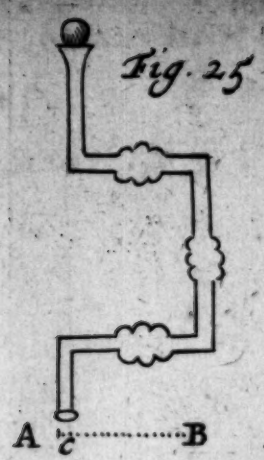


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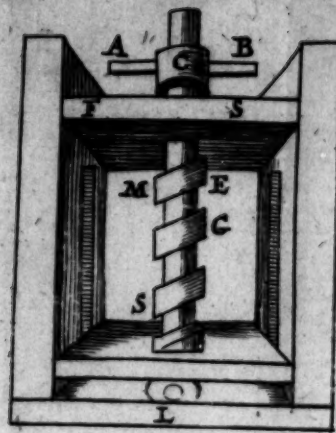


Fig. 33

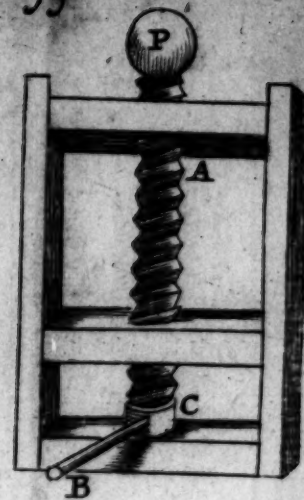


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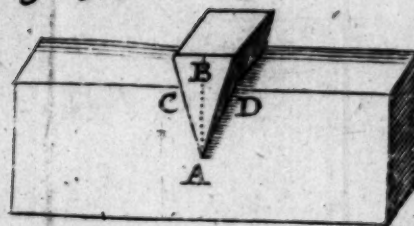


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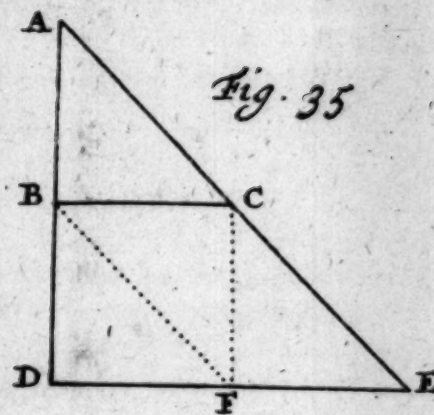


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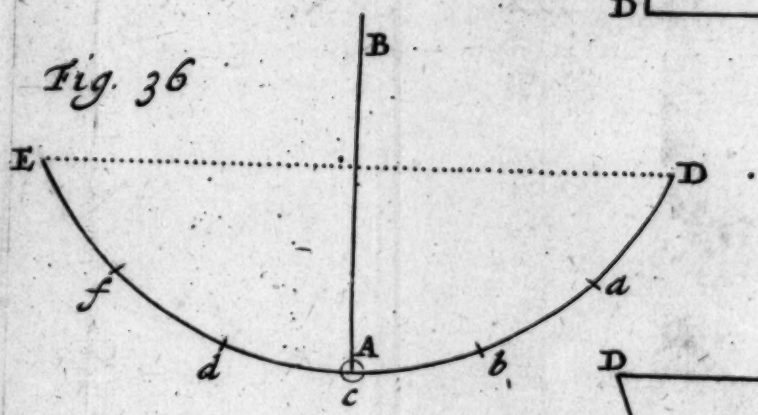


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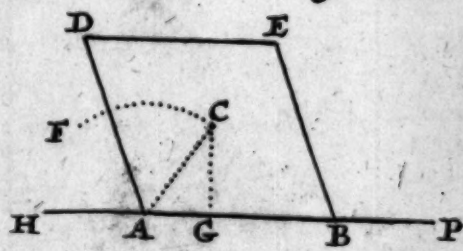


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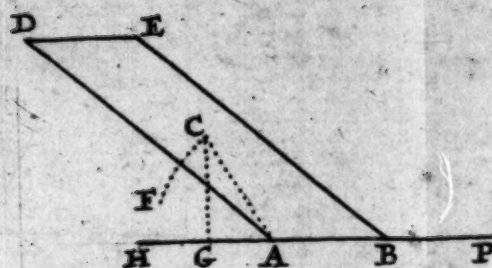


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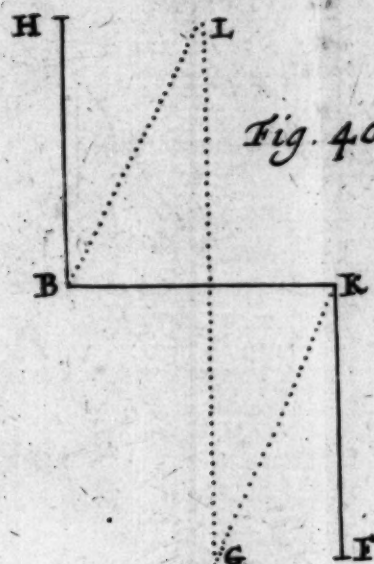
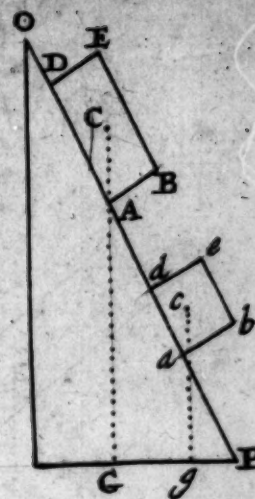


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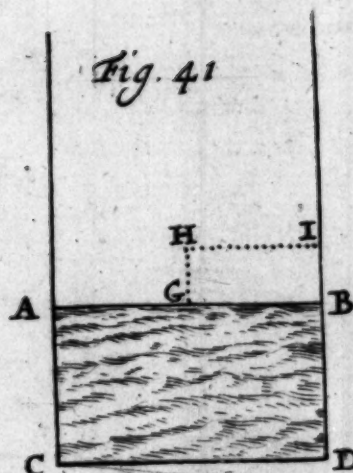


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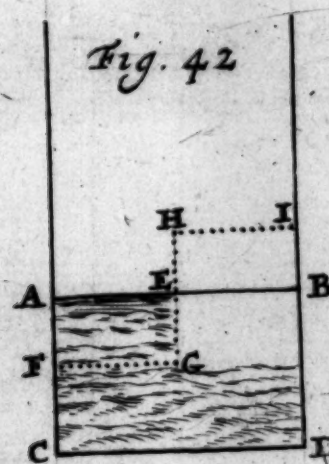


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Fig. 43



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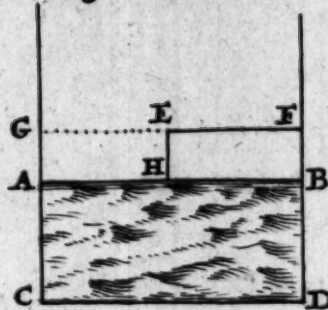


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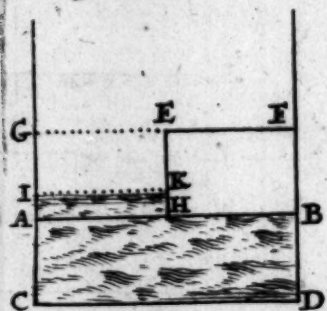


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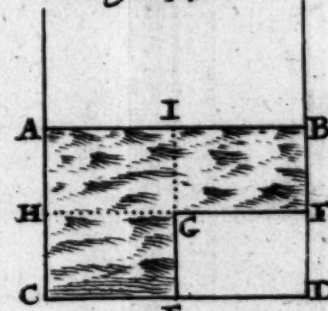


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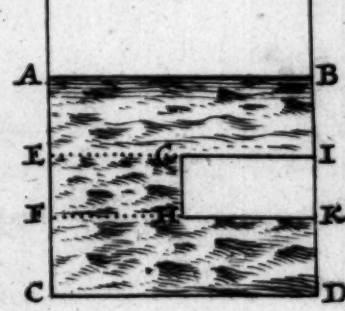


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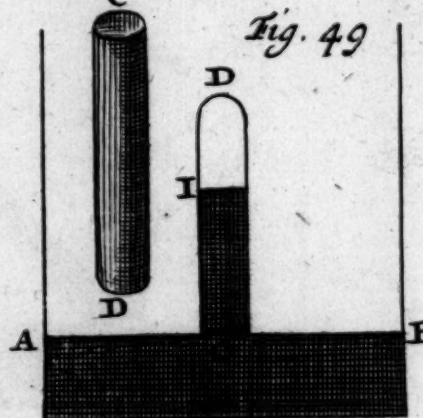


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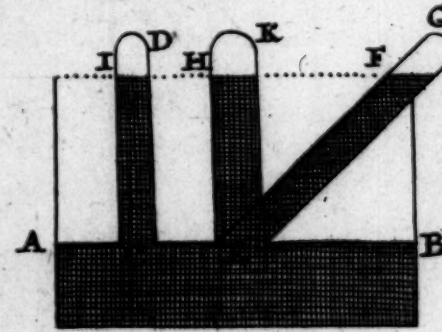


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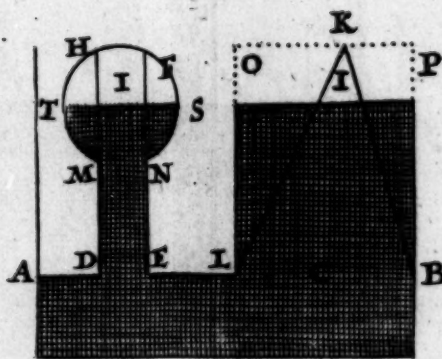


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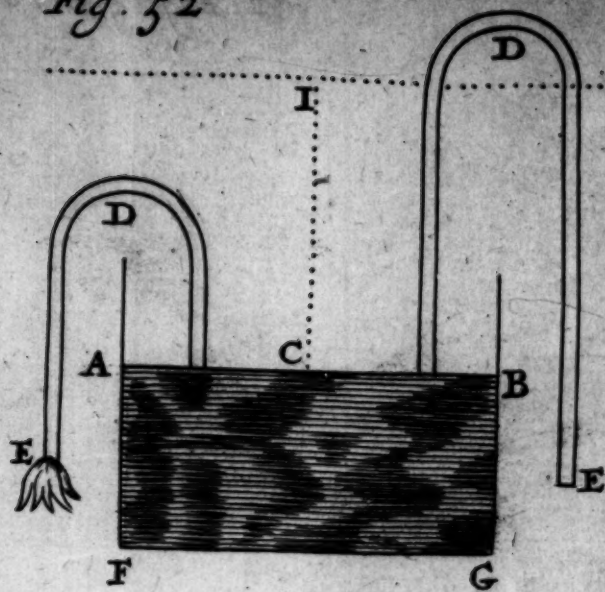
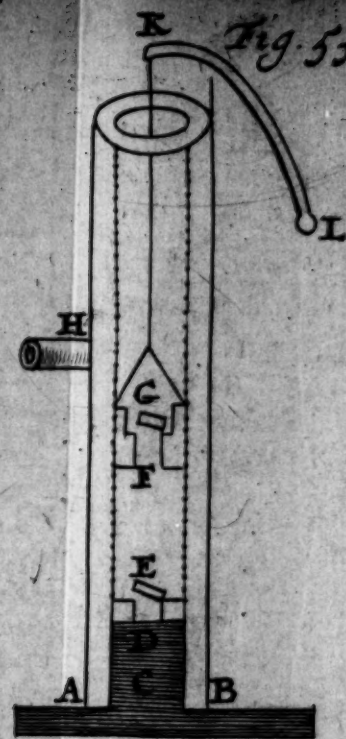


Fig. 53





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THE
Young Gentleman's
OPTICKS:

CONTAINING
The more Useful and Easy
ELEMENTS of *Opticks*, more
properly so called, of *Catoptricks*,
Dioptricks, and *Perspective*.

By EDWARD WELLS, D. D.
Rector of *Cotesbach* in *Leicester-*
shire.



L O N D O N,
Printed for *James Knapton*, at the *Crown*
in *St. Paul's Church-Yard*. 1713.

THE
Young Gentlemen's
OPTICKS

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The most Useful and
ELEMENTS of Opticks,
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Dioptricks, and Catoptricks.

By EDWARD WILKES, D.D.
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LONDON
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Opticks
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**THE
PREFACE.**

AS *Vision or Sight is the most useful and pleasant Sensation of the Body, so Opticks must consequently be very useful and pleasant, as not only explaining the Manner of Vision in a Geometrical Way, but also by the same Principles teaching how to supply or remedy the several Defects of the Sight, by the Help of proper Instruments, such as Spec-*
(F) *tacles,*

The Preface.

tacles, Microscopes, Telescopes, &c.

Add hereto, that to this Science is owing the Invention of several Instruments, which serve to please and divert the Mind by curious and surprizing Operations ; such as are the Dark-Chamber, the Magick-Lantern, &c. All which are accounted for in the following Treatise.

T H E

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THE
Young Gentleman's
OPTICKS.

CHAP. I.

Of Opticks in general.

THE Word *Opticks*, as it literally signifies in the Greek Tongue any Thing that relates to the Sight; so in its largest Acceptation it is used by the Learned, to denote the whole Compass of Science or Knowledge (Physical as well as Mathematical) relating to Vision or Sight.

It is now agreed by the Learned, that Vision is caus'd by the Eye's receiving into it Rays, which issue from the Object seen. These Rays, if they pass from the Object to the Eye through a Medium all

(F 2)

along

1.

Opticks in general, what.

2.

Opticks properly so call'd, what.

along of the same Nature, diffuse themselves all along after the same Manner by right (because the shortest) Lines. Whence this Sort of Vision is call'd *Direct Vision*; and the Word *Opticks* is taken in a more restrain'd Sense, to denote that Part of Opticks in general, which treats of direct Vision.

3.
Catop-
tricks,
what

If the Rays come not directly from the Objects to the Eye, but fall first upon some impenetrable Body, and are thence reflected or beaten back to the Eye, then the Vision is called *Reflex Vision*; and not the Object it self, but its Image is seen. A rough Surface so breaks and scatters the Rays, that they render no Image at all, or at best but a very imperfect Image, of the Object; and therefore those Bodies, which are capable of being most polish'd, or made most smooth, make the best Reflexion. Such is Glass, us'd therefore now-a-days for Looking-glasses; but Glass being of it self pellucid or transparent, there is usually put, on the Back-side of Looking-glasses, Quick-silver to render it impenetrable by the Rays. Before Looking-glasses were invented, the Ancients made use of polish'd Metal; and such a Piece of Metal, made use of to see one's self in, the *Greeks* call'd *Κάππτρον*; and from thence that Part of Opticks in general, which treats of reflex

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reflex Vision, is distinguish'd by the Name of *Catoptricks*.

If the Rays of the visible Object pass through different Mediums, so that they are more or less diffus'd in the one than in the other, then each Ray will be refracted or broken in one Medium, from that right Line wherein it diffus'd it self in the other Medium; whence this is called *Refracted* Vision. Thus a Ray passing out of Water into the Air appears refracted, i. e. broken or bent, from that right Line it made in the Water. And because the Knowledge of refracted Vision is most useful in order to the making of Telescopes, and such other Instruments; by looking through which, Objects are more clearly seen; hence that Part of Opticks in general, which treats of refracted Vision is distinguish'd by the Name of *Dioptricks*, *Διοπτρικὴ* being a Word capable of denoting in the Greek Language any Instrument made for to *look through*.

4.

Dioptricks
what.

To the foremention'd Parts of Opticks in general, may be added *Perspective*, or the Art of representing visible Objects in a Picture, just as they appear to the Eye. This Art is so call'd, because the Picture is to be suppos'd between the Eye and the Object, and transparent that so the Object may be seen through it; the Word *perspicere* in the Latin

5.

Perspective,
what.

Tongue signifying to see through a Thing.

I shall speak of each of the forementi-
on'd several Parts of Opticks, so far as
is requisite to the Design of this Trea-
tise; after that I have here adjoin'd the
Definitions or Explications of such Terms
or Words, as are of principal Use in Op-
ticks; as also such Theorems, as are a-
greeable both to the Principles of Geo-
metry and Experience, and therefore are
us'd as Axioms in demonstrating the The-
orems of this Treatise relating to Catop-
tricks and Dioptricks.

DEFINITIONS.

1. *A Radiant* is whatever sends forth
Rays from its several Points. Hence not
only the Object, suppose a Candle, is
called the Radiant; but also its Image,
suppose the Image of a Candle seen in a
Glass, is call'd so likewise. As is also
the Image of an Image, and so on.
Where note, that for Distinction's Sake,
the Object is usually specify'd by the
Name of the *prime* (or *primary*) Radiant.
And so the immediate Image of an Ob-
ject may be distinguish'd by the Name of
a *second* Radiant; the Image of the said
Image, by the Name of the *third* Radi-
ant, and so on; that which is seen by
the Eye being always distinguishable
(where

(where there are many Images) by the Name of the *last* Radiant.

2. *Parallel Rays* are such, as keep all along at an equal Distance one from the other.

3. *Diverging Rays* are such, as being produced both Ways, concur or meet in one Point that Way, which is contrary to their Motion. Thus *Fig. 10*, the Rays AD, AB, and AF, which are suppos'd to come from A, are diverging Rays.

4. *Converging Rays* are such, as being produced both Ways, concur the same Way they move. Thus *Fig. 11*, the Rays ED, OB, and HF, which are conceiv'd to move toward *a*, are converging Rays.

N. B. Parallelism, Divergency, and Convergency are to be understood of Rays of the same Point.

5. The *Focus* is that Point, where the Rays of the same Point being produced concur. Thus *Fig. 10*, A is the Focus of the diverging Rays AD, AB, and AF; and *a* is the Focus of the converging Rays, ED, OB, and HF. Whence it is obvious, that the Focus of parallel Rays must be conceiv'd to be at an *infinite* Distance, *i. e.* at so great a Distance, as that the Rays of the same Point, though they are strictly diverging or converging, yet diverge or converge so very little, (at least in respect of that Length and Porti-

on of them, wherein they are consider'd,) as that they may be look'd upon as parallel.

6. The *Angle of Incidence* is naturally and so properly the Angle ABC (*Fig. 8.*) comprehended between the incident Ray AB and the Surface BC which it (incides or) falls upon. But the Angle ABC being always the Complement of the Angle ABP to a Right; and because what is to be prov'd of the Angle ABC may be primarily, and so more easily prov'd, of the Angle ABP, hence for Convenience sake Writers of Opticks do usually take ABP for the Angle of Incidence, and agreeably define it to be the Angle comprehended under the incident Ray AB, and a right Line PB perpendicular to B the Point of Incidence. Agreeably also hereto,

7. A *Reflex Angle* is that, which is comprehended under the reflex Ray BE and the foresaid Perpendicular PB, viz. the Angle EBP *Fig. 8*: Though more properly or naturally EBG be the Angle of Reflexion.

8. A *Refracted Angle* is that, which is comprehended under the refracted Ray BG and the Perpendicular KF or BF, (*Fig. 9.*) viz. the Angle GBF.

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9. By *Inflexion*, as a general Word, is denoted either Reflection or Refraction. And consequently both the reflecting Surface BC (*Fig. 8*) and the refracting Surface BD (*Fig.*) are stiled inflecting Surfaces. And both a reflected Ray BE (*Fig. 8*) and a refracted Ray BG (*Fig.*) are comprehended under the Name of inflected Rays.

A X I O M S.

1. A Ray of Light falling perpendicularly on an inflecting Surface, either goes forward in a right Line, or is reflected into its self, Thus (*Fig.*) the Ray AC is to be conceiv'd as reflected by the reflecting Surface BD into it self. And the Ray AD (*Fig.*) is to be conceiv'd to pass forward beyond the refringent Surface BD in a right Line DE or DN.

2. The Inflexion of any Ray is performed in a Surface perpendicular to the inflecting Surface. Thus (*Fig.*) the Ray AB is reflected into BE in the Surface BCDE perpendicular to the reflecting Surface BC. And likewise (*Fig.*) the Ray AB is refracted into BG in the Surface BGND perpendicular to the refringent Surface BD. And note, that the said Surface wherein any Ray is inflected,

The Young Gentleman's

reflected, is thence stil'd the *Plane of In-*
flexion.

CHAP. II.

Of Opticks properly so call'd, or
Direct Vision.

THEOREM I.

Fig. I.

THE Eye is always placed in the
Center of its own Sphere of Vi-
sion.

Demonstration. At what Distance so-
ever, suppose Oa , the Eye O can see one
Way, it is self-evident, that the same
Eye, if by no means hinder'd, can see at
the same Distance any other Way; viz.
towards b it can see at the Distance Ob ;
towards c at the Distance Oc , and so
quite round. Wherefore the several Di-
stances Oa , Ob , Oc , &c. being equal one
to the other, and all proceeding from the
same Point O , it follows that they are
the Rays of a Circle, whose Center is O ,
and Circumference $abcdefgh$. Whence al-
so it follows, that the said right Lines Oa ,
 Ob , Oc , &c. are the Rays likewise of a
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Sphere generated by the Turning of the foremention'd Circle round about any of its Diameters, suppose ae , as an Axis; and consequently that O the Eye will be the Center of the said Sphere. Q. E. D.

Corol. 1. As it is evident from what has been said, that could the Eye be placed where it might see every Way, the Space, which would then fall within its Sight, would represent an entire Sphere; so it follows, that by the Interposition of the Earth, the Space or Compass of actual Vision, even when the Eye has a full View, is equal but to an Hemisphere. Namely, supposing ae to represent the upper Surface of the Earth whereon the Eye is placed, the Extent of actual Vision will be equal to the Hemisphere $Oabcde$, the Line EF denoting the Horizon or the Boundary of the Sight downwards.

Corol. 2. From this Theorem it follows also, that whatever Portion of the Heavens, (whether All $abcde$, or only Part of it, as bd , or bc , &c.) is seen by us, it appears as a Portion of the concave Surface of a Sphere.

Corol. 3. Because the Sun and Moon and Stars are beyond the Extent of distinct Vision, and consequently the Eye can't discern, how far one is more distant from it than the other; hence it comes to pass,

pass, that the Sun, Moon, and Stars, do appear to us, as all situated in the Surface *aceg* of one and the same Sphere *Oaceg*, and so equally distant from us, though they are in reality at several Distances from us.

Corol. 4. So much of the Heavens as is seen by us, *viz. abcd*, appearing of a spherical Figure only for the Reason contain'd in this Theorem; it thence follows, that the Body of the Heavens may be of a Quadrilateral Figure, as *ABCD*, or of any besides a Spherical.

Scholinum. This first Theorem with its Corollaries is of good Use to prepare young Students for Astronomy. And I have chosen to place this Theorem under direct Vision, partly because it will so be the more easily understood, and partly because, (though it be not one Homogeneous Matter contain'd in the Expanse or Space which we call Heaven, and consequently what we see therein at any considerable Distance, we see not by direct, but refracted Vision, yet) it comes to the same in respect as to the Case before us, whether Things in the Heavens are visible to us by direct or refracted Vision.

THEO-

THEOREM II.

The Eye can see at one View (*i. e.* at *Fig. 2.* one Time and Place) no more Objects, than whose Rays fall within the Compass of a Semicircle, to which the Eye, or rather its Pupil, is the Center.

Dem. For the Eye *ocaf* can see only those Objects, whose Rays, passing thro' *O* its Pupil, reach to its Retina or Optick Nerve placed at the Bottom of the Eye. But now it is evident from the bare Inspection of *Fig. 2.* that no Ray can enter the Pupil *O*, but what falls within the Semicircle *DHE*, whose Center is *O*. For supposing a Ray to pass along *ED* the Diameter of the Semicircle, it will pass over the Pupil *O*, and not enter it. And consequently all Rays falling without the said Semicircle, will much less enter the Pupil. Q. E. D.

Corol. Rays of Light (as well as other Things, whether natural or artificial) having a greater or less Force, as they are less or more oblique, it follows that such Rays as pass through the very Middle of the Semicircle *DHE*, are of most Efficacy in causing Vision, as entring the Pupil of the Eye perpendicularly, and consequently falling perpendicularly upon the Retina at the Bottom of the Eye. Thus
the

the Image of the Object A will be painted most lively and distinctly on the Retina. The Image *b* and *g* of the Objects B and G will be painted less lively and distinct than the former Image *a*, as being made by oblique Rays : but alike lively and distinct in respect of one another, as being made by the Rays *Bb* and *Gg* equally oblique : and more lively and distinct than the Images *c* and *f* of the Objects C and F, because the Rays *Bb* and *Gg* are less oblique than the Rays *Cc* and *Ff*.

Schol. Agreeably hereto we find by Experience, that we see most distinctly such Objects, as (*ceteris paribus*) are exactly before our Eyes. And upon this Account it is, that, in all the several Parts of Opticks largely so called, principal Regard is to be had to such Rays, as enter the Eye perpendicularly.

T H E O R E M III.

Fig. 2.

An Object appears greater or less, according as it is seen under a greater or less Angle.

Demonst. Let the Lines AB and CD (as they are of equal Length, so) represent two Objects of equal Bigness. It is evident that the Angle AOB, under which is seen the Object AB, is only a Part of the Angle COD, under which is seen the Ob-

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Object CD. It is also evident, that the Rays AO and BO being extended beyond O the Pupil of the Eye ODC will fall respectively upon the Points *a* and *b* of the Retina in the Bottom of the Eye; and likewise the Rays CO and DO being extended beyond O, will fall upon the Points *c* and *d* of the Retina. Whence it follows, that *ab* the Image of the Object AB will be so much in proportion less than the Image *cd* of the Object CD, that is, the Object AB will appear to a Spectator so much less than the Object CD, (though they be both of an equal Bigness in reality,) as the Angle AOB is less than the Angle COD.

Q. E. D.

Schol. Hence it appears, why the Distance between the same two Objects A and B appears to be greater or less, according as the Distance of the Spectator from the said Objects is greater or less; namely, because the Distance AB between the said two Objects is seen under an Angle proportionably greater or less, as the Distance of the Eye from the said Objects is lesser or greater; that is, the Angle AOB is proportionably greater than the Angle AEB, as the Distance OD is less than the Distance ED. And consequently (according to this *Theorem* III.) the Distance AB will appear so much in proportion

Fig. 4.

portion greater to a Spectator at O, than at E.

THEOREM IV.

Fig. 5.

The Rays of an Object being admitted through a narrow Hole, do therein cross one the other, so as that the Image made by the said cross Rays will be inverted in respect of its Object.

Demonst. The Ray *Aa* proceeding from the Point *A* of the Object or Arrow *AB*, being (in direct Vision) a right Line, and so likewise the Ray *Bb*, they must cross one the other, being extended beyond the Hole *O*. And therefore the upper Point *A* of the Arrow *AB* will become the lower Point *a* of the Image *ab*; and the lower Point *B* of the Arrow *AB* will become the upper Point *b* of its Image *ab*. And the like will hold as to all the intermediate Points, (except the very middle Point *C*, which being perpendicular or in a right Line with *O*, its Image *c* will likewise be the middle Point of the Image *ab*) and consequently the Image *ba* will be inverted in respect of its Object *AB*.
Q. E. D.

Schol. 1. Agreeable hereto are Experiments of this Nature. Namely, a Room being darken'd by putting together its Window-shuts, and a small Hole being made with a common Gimblet or Nail-

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passer in one of the Window-shuts, and the Glass of the Window being removed from before the Hole by opening the Casement or the like; such Objects as are without, and so situated as that their Rays will pass through the Hole, will appear upon a Sheet of white Paper, held on the Inside over against the Hole, in an inverted Manner according to this Theorem. And the nearer the Paper is held to the Hole, so much less will the Image be; because as much less as the Rays Aa and Bb are extended beyond O the Hole or Point of their Meeting, so much less is the Distance between them: Namely, the Distance cd is less than the Distance ab , and consequently the Image of the Object (suppose the Arrow) AB will appear less on the Paper held at the Distance cd from the Hole, than if the Paper be held at the Distance ab .

2. And the same has been found, by Experiments made for this Purpose, to hold good in respect of the Eye it self, this being as it were a small dark Chamber, into which are let pass through the Pupil the Rays of outward Objects, which paint their Images on the Retina in the Bottom of the Eye, agreeably to this Theorem, that is, in an inverted Manner with respect to its Object. As is il-

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lustrated

Illustrated *Fig. 6*, which represents an Eye having on its hinder Part a little Piece of each of its Coats or Skins cut away, 'till you come to the vitreous Humour. A Candle being held before the Eye, and a Piece of white Paper being apply'd to its hinder Part prepared as aforementioned, the Image of the Candle has been found to be made on the said Paper in an inverted Manner; All other Rays of Light being kept from falling on the Paper by dark'ning the Room where the Experiment was made. Concerning which more shall be said in Chapter IV, forasmuch as the Rays passing through the Humours of the Eye are refracted, and consequently it is requisite to understand the Laws of Refraction, before we can understand aright, how Vision is caus'd in the Eye. And for the same Reason I omit here saying any thing of the Use of spherical Glasses in the Hole to a dark Chamber, for rendring the Images more lively and distinct.

3. I shall add here (as being of great Use for the clearer Understanding of what has been or shall be said in this Treatise of Opticks) a Description of the several Parts of an Eye, represented *Fig. 7*, where A denotes the *Chrystalline* Humour, B the *Vitreous*, C the *Aqueous*: D the *Cornea Tunica*; E the *Retina*; F the Coat

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Coat call'd *Choroides* ; G the Coat call'd *Sclerotica*, being a Part of the *Dura Mater* continued about the Ball of the Eye on its inner Part ; H the Optick Nerve without the Ball of the Eye ; I the Coat or Skin call'd the *Uvea* ; K the Hole of the *Uvea*, call'd in one Word the *Pupil*.

It remains only to observe further, that most, if not all the Particulars taken notice of in this Chapter, belong not to direct Vision alone, but are applicable also to reflected and refracted Vision ; which we proceed to speak of in their Order.

C H A P. III.

Of Catoptricks, or Reflex Vision.

T H E O R E M I.

IF a Ray AB of Light be reflected by a plain Surface GC, the Angle PBE of Reflexion will be equal to the Angle PBA of Incidence. Fig. 8.

Demonst. Draw BP, and also AC, perpendicular to GC. The Medium GCde being suppos'd homogeneous to (*i. e.* of the same Nature with) the Medium EBCD,

(G 2)

EB $\overline{C}D$, it follows that all the Rays comprehended between AB and AC, and which take up or spread all over the right Line BC, were they not hinder'd (by GC the reflecting Plane) from passing thro' the Medium GC $\overline{d}e$, would therein diffuse themselves after the same Manner, as in the other Medium EB $\overline{C}D$, that is, (the right Line AB being continued to e , and AC to d ,) at the Distance Be they would diffuse or spread themselves all over de . And whereas the reflecting Plane BC does neither increase nor diminish the spreading of the Rays, but only turns them back into the same Medium they were in afore; hence it is obvious, that after the Reflection the Rays will spread themselves in the same Manner, as they would have spread, had they not fell upon the Surface GC, but (that being remov'd) gone on in the same homogeneous Medium. But in that Case, at the Distance Be, the Rays would (as is afore shewn) spread themselves all over ed . Wherefore in the Distance BE=Be, they will spread themselves all over DE=De: and consequently the Figure BCDE will be in all Respects like and equal to the Figure BC $\overline{d}e$; and indeed is no other than the Figure BC $\overline{d}e$ reflected, or turn'd about (as it were) half round upon BC. Wherefore, if from the equal Angles EBC and

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if $\angle BC$ there be taken away the right Angles $\angle PBC$ and $\angle pBC$, there will remain the Angles $\angle EBP$ and $\angle eBp$ equal one to the other. But $\angle ABP = \angle eBp$, therefore $\angle ABP = \angle EBP$.

Q. E. D.

Corol. 1. Hence the Angles $\angle EBG$ and $\angle ABC$ (which, as has been *Defin. 6* and *7* observ'd, are more properly call'd the Angles of Incidence and Reflection) are equal one to the other. Namely, because $\angle PBG = \angle PBC$, and $\angle PBE = \angle PBA$, therefore $(\angle PBG - \angle PBE =) \angle EBG = (\angle PBC - \angle PBA =) \angle ABC$.

Cor. 2. If the reflex Ray BE be taken for the Incident, then the incident Ray AB will be its Reflex.

Cor. 3. If the Angle $\angle PBE = \angle PBA$, or $\angle GBE = \angle CBA$, then BE is the Reflex of AB .

Schol. What has been prov'd as to a reflecting Plane GC , the same holds good also in respect of any reflecting *Curve* Surface PQ . For the Inclination or Direction of the Curve, at B the Point of Incidence, is the same with the Inclination of the Plane DE that touches it there. And therefore the Reflection following always the Inclination, will be the same, whether it be suppos'd to be made by B as a Particle of the Curve Surface PBQ , or of the Plain DBE . And the same holds good as to refracting Surfaces.

Fig. 9.

(G 3)

THEO.

THEOREM II.

Fig. 10. At what Distance CA an Object A is placed before a reflecting Plane CF (whether of Metal or Glass, &c.) at the same Distance Ca will a the Image of the Object appear behind the said reflecting Plane, each Distance CA and Ca being measur'd upon one and the same Perpendicular to the said Plane, *viz.* upon Aa .

Demonst. Let AD , AB , and AH be Rays proceeding from the Object (or any Point of the Object) A ; and let DE , BO , and FH be their respective reflex Rays. It is evident, that these being extended beyond CF (*i. e.* behind the reflecting Plane) will all meet in a , which therefore will be the Focus of the said Rays, and consequently the Place where the Image of a will appear. But now the Angle $aDC = EDF$, and $EDF = ADC$ by *Theorem I.* of this Chapter; wherefore $aDC = ADC$. And because in the Triangles ACD and aCD , the Angle $ADC = aDC$, and also $ACD = aCD$, and the Side CD is common to both Triangles, therefore $CA = Ca$. And after the very same Manner may it be prov'd, that in the other Triangles, ACB and aCB , as also ACF and aCF , the Distance

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stance CA of (any Point in the Object, and (*) consequently of all) the Object A is equal to the Distance Ca of the Image

a. Q. E. D.

Corol. 1. Hence it follows, that supposing the reflex Ray OB to be that which comes to the Eye plac'd at O, though all the rest of the Looking-glass or other reflecting Plane, CF be cover'd (or taken away) but the Part B, yet the Image a will be seen by the Eye; whereas if the Part B be hid, and all the rest uncovered, the Image can't be seen by the Eye at O.

Cor. 2. After the same Manner as Objects are inclin'd towards the reflecting Plane, will also their Images be inclin'd, And consequently in such a Plane or Looking-glass plac'd horizontally, Objects will appear inverted, or with their upper Parts downward.

Cor. 3. Because $aB = AB$, therefore $aO = AB + BO$, that is, the Distance of the Image from the Eye is equal to the

(*) Namely, as the visible Object is conceiv'd to consist of several Radiant Points, so the Image of the whole Object is made up of all the Images of the several Radiant Points, taken together; as is more fully illustrated by Fig. 11 and 12, of which see more in the following Scholium.

The Young Gentleman's

incident Ray and reflected Ray taken together.

Cor. 4. Whatever has been said of the Image of any Object or prime Radiant, the same holds true also of the Image of an Image. Whence arises the multiplying of Images by Means of two or more reflecting Planes duly placed. Wherein this is most remarkable, the Distance of every Image is equal to the Ray propagated from the prime Radiant to the Eye, through all the intermediate Reflexions.

Schol. 1. It is observable, that by this Theorem, the Focus A of Rays AD, AB, and AF falling on the reflecting Plane CF being given, thereby may be found the Focus *a* after Reflection; namely, by drawing AC perpendicular to CD, and extending it to *a* so, as that Ca may be equal to CA. For then *a* will be the Focus sought.

Schol. 2. For the Sake of such as may not so readily apprehend from *Fig. 10*, how the Image of a whole Object is seen by Reflection, I have therefore added *Fig. 11* and *12*. In each of which AB denotes a Cross, and *ab* its Image seen by the incident Rays reflected to the Eye at E by the reflecting Plane CF. Namely, *Fig. 11* shews expressly, how the Rays AG and

and BI that come from the two Extremities aB of the Object are reflected to the Eye, and so cause the Image of A and B to appear at a and b ; and *Fig. 12* shews how in like Manner the Images of the Points D and H of the Object appear at d and h ; namely, in both Figures the Images appear, where the reflex Rays EG and EI being extended, meet with the Perpendiculars Aa , Bb , Hh , and Dd ; and where $Ca=CA$, $Fb=FB$, $Ch=CH$, and $Fd=FD$. And the same may be shewn as to any other Points.

THEOREM III.

If a reflecting Glass (or other Surface) *Fig. 13.*
 BD be spherical, either concave or convex, and the Object E be far distant from the Glass; its Image will, to the Eye placed in the Axis AB of Radiation, appear at C the Middle of the Semidiameter AB of the Glass. But if the Object E be not far distant, then its Image *Fig. 14, 15, 16.*
 will, to the Eye in the Axis, appear at that Place C, where $(BC : CA :: BE : EA$, that is, where) the Distance of the Image from B the Vertex of the Glass will be to its Distance from A the Center of the Glass, as the Distance of the Object from the said Vertex is to its Distance from the said Center.

I have chosen to comprehend all that relates to reflecting spherical Glasses, with respect to the Eye placed in the Axis of Radiation, under one Theorem, that thereby may the more easily be discern'd the uniform Manner, whereby Vision is perform'd in such Cases.

In reference to which it is to be observ'd, that (although in the Figures belonging to this Theorem, the Point D be placed at some considerable Distance from B, for the more distinctly perceiving the Demonstrations, yet) D is to be suppos'd very near to B, because otherwise the Reflex CD of the Incident Ray ED will not affect the Eye placed in AB. For the reflex Rays of all such Rays as fall more remote from B, will pass by on one Side of the Pupil of the Eye, and so contribute nothing to the Sight of the Image. Add hereto, what has been observ'd in the *Corollary* of *Theorem* 2, Chap. 2, Hence Notice is here taken only of such Rays as fall upon the Glass very near to its Vertex B. These Particulars being premis'd, I come now to the Demonstration of this Theorem, in respect of its four principal Cases following.

Fig. 13.

Case 1. When the Object is far distant from the Glass, so that the Rays coming from any one Point thereof may be conceiv'd parallel one to the other, as well

as

as to the Axis AB, as the Ray ED. In such a Case the Image will appear, to the Eye placed somewhere in the Axis, at C the Middle of the Semidiameter of the Glass. For bisect AB in C, and draw AD and CD, and produce them on the other Side of BD the Glass. Because the Point D is look'd to be so very near to B, as in a Manner to coincide with it, therefore we may also look upon $CD=CB$; and by Construction $CA=CB$, therefore $CA=CD$, and consequently the Angle $CAD=CDA$. But the Angle $CAD=EDA$ the Angle of Incidence of the Ray ED, (for AD is perpendicular to the Surface of the spherical Glass,) wherefore the Angle CDA is equal to the Angle of Incidence of ED, and therefore (by *Cor. 2, Theorem I. Chap. 3.*) DC is the Reflex of the Incident ED upon the Concave Surface of the Glass BD. And the like may be demonstrated as to the Convex Surface of the Glass BD. For, because the Angle $EDA=eDO$, and $CDA=NDO$, therefore the Line DN will be the Reflex of the Ray eD, and being produced backwards (or on the Concave Side of the Glass) will touch AB also at C. And what has been demonstrated of any Ray ED taken at pleasure, the same holds true of all Rays in the same Circumstances. Wherefore Rays
parallel

parallel to AB, and whose reflex Rays contribute to the Sight of the Image, if they fall upon a Concave Glass, after their Reflexion will concur at C; and from thence diverging again, will render the Image visible at C, to the Eye placed somewhere in AB. And the reflex Rays of such parallel Rays, as fall upon a Convex Glass, do diverge from C, and consequently render the Image visible at C, to the Eye plac'd in AB produc'd on the convex Side of the Glass, that is, if eD be the incident parallel Ray, ND will be the Convex, which will form the Image at C. Q. E. D.

Corol. Hence, and from *Corol.* 2. *Theorem* I. of this Chapter, it follows, that Rays diverging from C and reflected by a Concave Surface, or converging at C and reflected by a Convex Surface, are parallel to AB; that is, if CD be the Incident, DE is the Reflex; or if ND be the Incident, De is the Reflex.

Fig. 14. *Case* 2. If the Object E be not far distant from BD the Glass, and so the Rays proceeding from any single Point thereof are (not parallel, but) diverging, then the Image will, to the Eye in the Axis, appear at that Place C , where $BC : CA :: BE : EA$. And this shall be demonstrated first in respect of Concave Glasses, and when the Distance of the Object is greater

greater than half the Semidiameter of the Glass. For let the Point C be so taken in AB, as is requir'd ; and draw AD and CD. Because (for the Reason above-mention'd) $ED=EB$, and $CD=CB$, therefore by Construction $CD (=CB) : CA :: DE (=BE) : EA$. Wherefore alternately $CD : DE :: CA : EA$. And so by (*Prop. 3. Euclid. 6.*) the Angle $ADE = ADC$. But ADE is the Angle of Incidence of the Ray ED ; therefore (by *Corol. 2. Theorem I.* of this Chapter) DC is the Reflex of the Ray ED. And ED being taken at Pleasure, it follows that the Focus of all other Rays diverging from E will, after their Reflection by a concave Glass, be at C, and consequently the Image of the Object will appear at C, to the Eye placed in the Axis (AB produced, or) EB. Q. E. D.

Cor. 1. Hence C will be the Focus of Rays ED converging to E in the like Circumstances, and reflected by a spherical convex Reflector.

Cor. 2. Hence and from *Cor. 2. Theorem I.* of this Chapter, if CD be the Incident, DE will be the Reflex ; and if ND be the Incident, De will be the Reflex.

Case 3. And the same holds true, when *Fig. 15.* the Distance of the Object is less than half the Semidiameter of the Glass. For
in

in AB extended, let C be taken according to the Prop. be requir'd ; and let AD and CD be drawn and produced ; as also let ER be drawn parallel to CD. The Arch BD being as Nothing, therefore $ED=EB$, and $CD=CB$. Whence $ED:EA::CD:CA$; and so (by Reason of DC being parallel to ER) $ED:EA::ER:EA$. Wherefore $ER=ED$, and the Angle $ERD=EDR$. But EDR is the Angle of Incidence of ED, and $ERD=NDA$ its Altern, therefore (by *Cor. 2. Theorem I.* of this Chapter,) DN is the Reflex of the Incident ED. And ED being taken at pleasure, it is manifest, that all Rays proceeding from E, and reflected by a concave Glass, being extended backwards, will concur in C ; and consequently the Image of the Object will appear at C, to the Eye placed in the Axis. Q. E. D.

Corol. 1. Hence if CD be the Incident on a spherical Convex Reflector, DE will be the Reflex, and so the Image will appear at E.

Cor. 2. Hence and from *Cor. 2. Theorem I.* of this Chapter, if ND be the Incident, DE will be the Reflex ; and if eD be the Incident, DC will be the Reflex.

Fig. 16. *Case 4.* The like holds good in respect of convex Glasses. For in AE let C be taken according to the Proportion requir'd

quir'd in this *Theorem*, or (which comes to the same) so as that $AC : CB :: AE : EB$, and let CD be drawn and extended; and parallel thereto let ER be drawn, 'till it meets in R with AD produced. The Arch BD being as Nothing, therefore $CD = CB$, and $ED = EB$. And therefore $AC : CD :: AE : ED$. But because the Triangles ACD and AER are equiangular, therefore $AC : CD :: AE : ER$; and so $AE : ED :: AE : ER$; and therefore $ER = ED$. And consequently the Angle (ERD , or its Altern,) and so Equal) $NDR = EDR$ the Angle of Incidence of ED . Wherefore DN is the Reflex of the Incident ED . But ED being taken at pleasure, it is evident, that all the Rays diverging from E , and entring the Eye placed in the Axis, after they are reflected by the convex Surface BD , will diverge from the Focus C ; and consequently the Image of the Object E will appear at C to the Eye so placed. Q. E. D.

Coroll. Hence if ND be the Incident, DE will be the Reflex; and if in a concave Reflector eD be the Incident, DC will be the Reflex; or if CD be the Incident, De will be the Reflex.

Schol. From this *Theorem* is deduced the Way of finding the Focus of parallel

lel or other Rays, falling upon spherical Surfaces, after their Reflection, viz.

The Focus C of such parallel Rays ED after Reflection is found by bisecting the Semidiameter AB of the Sphere in C.

And the Focus C of other Rays falling upon a spherical Surface, is found after Reflexion, by taking C so, as that $BC : AC :: BE : AE$.

And thus it has been shewn, both by Way of Theorem and Problem, where the Image of an Object will appear in any spherical Glass, (or other like reflecting Plane) to the Eye placed in the Axis of Radiation. And this is sufficient to our present Purpose, not only because most optical Instruments are made after this manner, but also because the Image, only in this Situation of the Eye, appears so lively and distinct, as to deserve the Name of an Image. Hitherto both Object and Image have been denoted only by single Points, as being sufficient to illustrate what has been afore deliver'd. I now proceed to speak of Object and Image consider'd as Lines or Surfaces.

THEO-

THEOREM II.

The Image CT of a radiant plain Surface FE, made by a spherical Glass BD, is also a plain Surface. Fig. 17
& 18.

From A the Center of the Glass, draw AE perpendicular to FE, and meeting the Glass in its Vertex B. And (according to *Theorem III*, of this Chapter) let C be the Image of the Point E, through which draw parallel to FE the Plane CT. In the Plane CT will appear the Image of the Plane FE made by the Glass.

Thus let AF be the Ray, drawn from any other Point F of the radiant Plane FE to the Center of the Glass, and meeting with the Glass in D, and the Plane CT in T. And by Construction let CD be the Reflex of the Incident ED; and let the Reflex of the Incident FB be BH, meeting the Ray FD in H. Now, because we suppose in the present Case the Angle FAE to be small, therefore the Arch BD will be also small, and in effect no other than a little or short right Line. And the Circumference of the Circle describ'd with the Diameter BF will pass through the Points D and E, because the Angles FEB and FDB are right. Whence the Angles BFD and BED being in the same Segment, are equal. And when the Angles at A are Verticals or the same,

(H) it

it will follow that $FBA = EDA$. And (because by *Theorem* I, of this Chapter) $ABH = FBA$, and $ADC = EDA$, therefore $ABH = EDC$. Whence the Triangles ABH and ADC are equiangular, and so similar. Wherefore $AB : AH :: AD : AC$; and whereas $AB = AD$, therefore $AH = AC$. But by reason of the Smallness of the Angle CAT , we may look upon $AT = AC$, and therefore we may also look on $AH = AT$. But H is the Image of the Point F in the radiant Plane, whence the said Image is plac'd in the Plane CT . And the same may be shewn the same Way as to the Image of any other Point in the Plane FE . Wherefore the Image of the Plane FE is placed in the Plane CT . Q. E. D.

If the radiant Plane be far distant, then the Images of several radiant Points will be (by *Theorem* III, of this Chapter) in the Mid-points of the several Semidiameters of the Glass tending to the said radiant Points, that is, the Image of a radiant Plane far distant will have the Figure of a spherical Surface concentrical to the Glass. But because the said Radiant is (according to our Supposition) seen under a small Angle, that small Portion of a spherical Surface, which is taken up by the said Image, will scarcely differ from a plain Surface, to which AB is perpendicular.

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Whence this Theorem holds good in all Cases : for the Demonstration will serve for any other Case, as well as those two represented in *Fig. 17* and *18*.

Cor. If the Angle *EAF* be too great, the right Line *AH* will be sensibly less than *AT*. Whence an Object being seen under such an Angle, its Image, if made by a concave Glass, will appear concave; if made by a convex Glass, will appear convex.

THEOREM V.

The Object *HF*, and its Image *bf*, are *Fig. 19* seen under equal Angles *HBF* and *bBf* & 20. from the Vertex *B* of the reflecting Curve *BD*, whether convex (as *Fig. 19*) or concave (as *Fig. 20*.)

Demonst. Draw *AE* perpendicular to *HF*, (which by *Theorem IV*, of this Chapter, will consequently be perpendicular to *bf*.) and meeting the reflecting Curve *BD* at *B*. Draw also *BF*, *BH*, *Bf*, *Bb*, and *Ff*, which last will pass through the Center *A*; forasmuch as according to our Supposition the Image of every Point is placed in the Axis of Radiation. In like Manner the Points *H*, *A*, and *b* will be in the same Axis. Now by *Theorem III*, of this Chapter, $BE : EA :: BC : CA$. Wherefore $BE : BC :: EA : CA$. But (by reason of the equiangular Triangles *AEF* and *ACf*) $EA : CA :: EF : Cf$. Wherefore

(H 2) fore

fore $BE : EF :: BC : Cf$ And whereas the Angles BEF and BCf are right, therefore the Angle $EBF = CBf$. And in like Manner the Angle $EBH = CBh$. Wherefore $HBF = hBF$. Q. E. D.

Corol. The Object FH (if it be a Line) and its Image fh are one to the other, as their Distances from the Vertex, BE and BC . But if the Object be a Surface, it and its Image will be one to the other, as the Squares of their Distances aforesaid. Whence, the Distance of the Object and the Image from the Reflector being given, thereby is given also the Ratio or Proportion of the Image to the Object. And in like Manner may be found the last Image of an Object, when it is propagated by many Reflecters. Lastly, it is evident, that the same will hold true in all respects, if fh be the Object, and HF the Image.

Schol. By what has been said, it is easy to account for and illustrate the Particulars relating to Images made by spherical Reflecters. Of which the more remarkable are these that follow, viz.

1. If the Reflector be concave, and the Distance BC of the Object hf from B the Vertex of the Reflector, be less than half the semidiameter, then Its Image HF will appear beyond or behind the Reflector, and in a right Position, (as in a plain Re-

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Reflector, namely the upper Part of the Object will appear the upper Part of the Image; the lower Part of that, the lower Part of this &c.) and as the Object is placed nearer or farther from the Reflector, within the Distance of. the Semidiameter, so will the Image be nearer or farther from the same, and appear less or more magnify'd. All which Particulars will be illustrated by comparing together *Fig.* 19 and 21 and 22. Namely in *Fig.* 21, the Object *hf* being placed a fourth Part *BC* of the Semidiameter *BA* from the reflecting Concave *BD* and in *Fig.* 19, the Object *hf* being placed a third Part *BC* of the Semidiameter *BA* from the same concave Reflector, it is evident that in the former the Image *HF* (as well as the Object *hf*) is nearer to the Reflector, and less proportionably, than in the latter. And in both Figures the Image appears in a due Position, *viz.* *F* is on the same Side of the Axis *AB* as *f* and *H* as *h*. And therefore if *f* be the upper and *h* the lower Part of the Object, *F* will be likewise the upper, and *H* the lower Part of the Image. Lastly, it is evident, that in both Figures *hf* and *HF*, *i. e.* the Object and its Image, are one to the other, as their Distances from the Vertex *BC* and *BE*; *viz.* in *Fig.* 19, $BC : BE :: 1 : 3 :: hf : HF$. And in *Fig.* 21, $BC : BE :: 1 : 2 :: hf : HF$. (H 3) 2. It

2. It is evident from comparing together *Fig. 19* and *22*, that if the Reflector be convex, and so HF the Object, and *hf* its Image, then likewise the Image will appear in its due Position; and also as the Object is brought nearer to the Reflector, so will the Image come nearer. But in this a convex Reflector differs from a concave, *viz.* that (whereas in a concave, the nearer the Object *hf* is to the Vertex B, the less will the Image HF appear, as is evident from comparing, as has been said, *Fig. 19* and *21*, on the contrary) in a convex, the nearer the Object HF is to B the Vertex, the greater will the Image *hf* appear, as is evident from comparing *Fig. 19* and *22*. It is also remarkable, that be the Object HF removed never so far distant from the reflecting Convex beyond the half Semidiameter, yet its Image will fix it self at the half Semidiameter.

3. If the Reflector be a Concave, and the Object HF be more distant from it than the length of the Semidiameter AB, (as *Fig. 20* and *23*) then the Image *hf* will be more distant from it than the half semidiameter, and yet less than the whole Semidiameter, that is, it will appear in the Air between the Object and the Reflector; and also it will appear in an inverted Position, that is, the Point F and its Image *f* will appear

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pear on contrary Sides of AB the Axis, as will the Point H and its Image h ; (and so of any other Point of the Object, and its Image,) whence if F be the upper or right Hand Part of the Object, f will be the lower or left hand Part of the Image, &c. Likewise it is evident from comparing Fig. 20 and 23, that in this Case, as the Object HF is remov'd farther from the Reflector, so its Image hf will come nearer and appear less; and on the contrary, as the Object comes nearer, the Image will go off farther, (and appear bigger) 'till they meet one the other at the Center A. In like Manner if fb be the Object, then HF will be the Image; the Affections whereof are easie to be accounted for, from what has been said. It is also observable that if the Object HF be remov'd so far from the Reflector, as that the Rays of it falling near the Vertex B of the Reflector, may be esteem'd parallel to AB, then (according to the former Part of *Theorem III*, of this Chapter) the Image hf will appear at the Mid point of the Semidiameter of the Reflector. And if the Sun be the said far distant Object, then the Place of the Image (or Mid-point of the Semidiameter) will be the *burning-point* of the Reflector, if it be considerably larger than the Image of the Sun. But if a lucid Body, as a Candle,

be plac'd in the said Mid-point of the Semidiameter, it will cast a Light to a great Distance. Add as in this Case, nothing but Light will appear on the Surface of the Reflector; so when other Objects are placed at the half Semidiameter, nothing but their Colour will appear on the Surface of the concave Reflector.

4. It is observable, that supposing the Object to stand still, and the Eye to change its Place, the same Alterations will be produc'd in respect to the Image, as if the Object had chang'd its Place in a correspondent Manner.

5. It is evident from *Fig. 19—23*, that a reflecting Concave magnifies, that a reflecting Convex lessens an Object. Namely, if in the said Figures BDG be consider'd as a Concave, then *bf* will be the Object, and HF the Image, which is in all the said Figures much larger than *bf* the Object. But if DBG be consider'd as a Convex, then HF will be the Object, and *bf* the Image, which is in all the said Figures much less than HF.

6 It is observable, that cylindrical Reflecters whether concave or convex, must render the Image deform'd. For a Cylinder being generated by the Motion of a right Line round two parallel Circles, hence such a Reflector must partake, partly of the Properties of a plain Reflector, partly

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partly of a spherical. Of that, in respect of its right Lines, in Longitude; of this, in respect of its circular Lines, Round-ways. Hence the Longitude or Height of an Object being represented justly, as by a reflecting plain Surface; but its Latitude magnify'd or diminished, according as the Reflector is a Concave, or Convex; the Image appears disproportionable. But on the same Account, if a Picture be drawn proportionably bigger in breadth than in Length, its Image will be represented by a reflecting Cylinder in its just Dimensions. Which is illustrated. *Fig. 24* 25, 26. Namely, in *Fig. 24*, is drawn the Picture in its just Dimensions, and divided into Parallelograms. In *Fig. 25* are drawn as many circular Lines, as there are (in *Fig. 24*) horizontal Lines parallel one to the other, viz. 7; all the said circular Lines being concentric to the circular Basis of the Cylinder C; and distinguish'd proportionably by right Lines drawn a-cross them into as many Divisions, as the Picture (*Fig. 24*) is divided into Parallelograms: So much of the Picture, as is contain'd in each Parallelogram being to be drawn in each of the foremention'd Divisions answering thereunto. Which being done, and the reflecting Cylinder being plac'd on C, the Image of the Picture will appear in the said

said Cylinder according to its just Dimensions, but less than the Picture, if the reflecting Cylinder be convex; which is generally us'd in this Case, and is represented *Fig. 26.*

Lastly, It remains only to observe, that by the foregoing Theorems may be found such a Position of the Radiant, as that the said Radiant may have a given Proportion to its Image made by the Reflector. For Instance, let *BD* (*Fig. 27*) be the Reflector, and the given Proportion be that of *BA* to *AM*. If the Object be a Line, bisect *BM* in *E*, and by *Theorem III*, of this Chapter find the Focus *C* answering to the Focus *E*, which *C* will be the Place of the Radiant. For because $BC : CA :: BE : EA$; therefore $BC : BC + CA :: BE : BE + EA$; that is (because $BE = EM$) $BC : BA :: BE : AM$; and therefore $BC : BE :: BA : AM$. But by the *Corollary* of this *Theorem V*, *BC* is to *BE*, as the radiant Line in *C* is to its Image in *E*; therefore the Radiant Line in *C*, is to its Image made by *BD* in *E*, as *BA* to *AM*, i. e. in the Proportion given. If the Radiant be a Surface, then instead of *AM*, a right Line must be taken, to which *BA* has a subduplicate Ratio of that which it has to *AM*. And this is sufficient to our Design, concerning Catoptricks or reflex Vision.

CHAP.

C H A P. IV.

Of Dioptricks, or Refracted

Vision.

T H E O R E M I.

IF a Ray AB of Light be refracted by *Fig. 28,*
 the plain Surface BD of a Medium of *29.*
 different Thickness, the right Sine of ABK
 the Angle of Incidence will, in respect of
 the same different Mediums, always have
 the same (Ratio or) Proportion to the
 right Sine of GBF the Angle of Refrac-
 tion, however the Ray AB be inclin'd to
 the Surface BD.

Demonst. Let BG be the Refract of the
 Ray AB, whether AB passes out of a
 thinner Medium into a thicker, (as *Fig.*
28) or out of a thicker into a thinner,
 (as *Fig. 29.*) Now as the Light proceed-
 ing from A, and contain'd between AB
 and AD, will be diffus'd within the Me-
 dium DK all over BD; so after it has en-
 ter'd the new Medium DG, it will at the
 Length of BG diffuse it self all over GN;
 Whereas had the same Medium DK been
 con-

continued on each Side of BD, the Light at the Length of BC ($=BG$) would have been diffus'd all over CE. But in both Cases (namely, whether the Medium DG be different from or the same with the Medium DK) $FN=ME$, and consequently both the said Lines may be look'd upon, as not at all depending on the (*) Facilities of the Medium's, and so of no Consideration in the present Case. Wherefore GF (*i. e.* $GN-FN$) and CM (*i. e.* $CE-ME$) are to be look'd on, as the genuine Effects of the Facilities of the two Mediums DG and (DC or which is the same) DK; that is, GF is to CM, as the Facility of the Medium DG is to the Facility of the Medium DC or DK. But CM is the right Sine of the Angle ($CBM=$) ABK, which is the Angle of Incidence of the Ray AB; and according to the same Semidiameter, GF is the right Sine of GBF the Angle of Refraction of the same Ray AB. Wherefore since the Angle DAB ($=ABK$) is taken at pleasure, the like will hold good of any other Angle; that is, in the Refraction of any Ray AB, let it fall how it will on the Plain whereby it is refracted, the Sine of the Angle of Incidence is to

(*) This Word is us'd to denote the several Virtues, which different Mediums have to diffuse the Light.

the Sine of the Angle of Refraction, as the Facility of the Medium DK is to the Facility of the Medium DG, whereby the Ray AB is refracted into BG. But in respect of the same Mediums, the Thicknesses of the Mediums, and the Facilities thence arising, and their Ratio will abide the same. Wherefore in the Refraction of any Ray, the Sine of the Angle of Incidence always keeps, with respect to the same Mediums, the same Ratio or Proportion to the Sine of the Angle of Refraction. Q. E. D.

Corol. 1. If BG be the refracted Ray of the Incident AB, the rest remaining the same, BA will be the refracted Ray of the Incident GB.

Cor. 2. The refracted Ray BG of any Incident AB is rightly assign'd, when the Sine CM (of the Angle CBM=) ABK the Angle of Incidence, is to the Sine GF of the Angle GBF contain'd under the assign'd Ray BG, and the right Line BM perpendicular to the refracting Surface BD, as the Facilities of the given Mediums are one to the other. Namely, this Corollary is no other than the Converse of this first Theorem.

Schol. 1. In a Ray's passing out of a thicker Medium into a thinner (as *Fig.*) if it be so inclin'd to the refracting Surface, that GF (which is to CM, as the
Faci-

Facilities of the given Mediums) does exceed the Semidiameter BC; then the incident Ray AB will have no refracted Ray, (after the Manner of an impossible Case in a geometrical Problem;) that which ought to be the refracted Ray not entering the thinner Medium DG, but being reflected by its Surface, according to *Theorem I.* of Catoptricks. And this Case is illustrated *Fig. 30.*

Schol. 2. When a Ray of Light passes out of the Air into Glass, it is observ'd that the Sine of the Angle of Incidence is to the Sine of the Angle of Refraction, as 3 to 2 in whole Numbers; and in passing from the Air into Water, I is to R, (by which Symbols henceforward is denoted the Ratio of the Sine of the Angle of Incidence to the Sine of the refracted Angle, which measures the Refraction) as 4 to 3. And on the other hand, in the passing of the Ray out of Glass into Air, I will be to R as 2 to 3; and in its passing out of Water into the Air, I will be to R as 3 to 4. Whence the Reason is manifest, why a Ray passing out of Glass, and falling on the Surface of the Air so, as to make the Angle of Incidence more than about 42 Degrees, will not enter into the Air, but is reflected by its Surface. Namely, because $2 : 3 :: 42 : 63$; that is, the Ratio sesquialtera (as it is called) of 42 is

63;

63 ; and consequently the right Line GF, which is to denote the said Ratio of the Sine of 42 Degrees must exceed the Ray, this being the Sine only of 60 Degrees. In which Case the Ray is reflected according to *Schol.* 1.

Schol. 3. It is demonstrable that if Rays diverging (*Fig.* 30 and 31) from E, and refracted by the Surface BD, do diverge from C, the Mediums being transpos'd, but the same intermediate Surface BD remaining, the Rays converging to E, will after Refraction converge to C. Wherefore what shall be demonstrated in the following Theorems concerning diverging Rays, may also be apply'd to converging Rays.

THEOREM II.

If an Object E be seen through different Mediums BDA and BDO, the Image of the said Object will appear, at that Place C in the right Line EB (perpendicular to the plain Surface BD of the refracting Medium, whether thicker or thinner) where $CB : EB :: I : R$, that is, where the Distance of the Image from the plain Surface of the refracting Medium is to the Distance of the Object from the said Surface, as the Sine of the Angle of Inci-

Fig. 30,
& 31.

Incidence is to the Sine of the Angle of Refraction.

Demonst. Let ED be any Ray of the Object E falling upon the Surface BD. Thro' D draw AO parallel to EB. Join C and D, and produce CD to F. Those Rays only being here (as well as in the foregoing Chapter) consider'd, that fall near EB the Axis of Radiation, and wherein the Eye is suppos'd to be plac'd, it follows (for the Reasons mention'd in the foregoing Chapter) that we may look on $ED = EB$, and $CD = CB$. Wherefore $CD : ED :: I : R$. But CD is to ED, as the Sine of the Angle BED or its Equal EDA, to the Sine of the Angle BCD or its Equal ODF. Therefore the Sine of the Angle EDA is to the Sine of the Angle ODF, as I to R. But EDA is the Angle of Incidence of the Ray ED, therefore ODF is the refracted Angle answering thereto; that is, DF is the refracted Ray of the incident Ray ED. And the same may be shewn of any other Ray diverging from E. Wherefore C will be the Focus of all Rays diverging from E and refracted by BD, that is, the Image of the Object E will appear after Refraction at C. Q. E. D.

Corol.

Corol. 1. Hence and from *Cor. 1, Theorem I*, of this Chapter it follows that (*cæteris paribus*) Rays in the Medium BDO converging to C, will after Refraction converge to E. And if the Mediums be transpos'd, then from hence and *Schol. 3, Theorem I*, of this Chapter it follows, that Rays in the Medium BDO converging to E, will, after Refraction, converge to C; and that parallel Rays will, after Refraction at the plain Surface of any Medium, still continue parallel.

Corol. 2. If BDA be Water, and BDO Air, then $CB : EB :: 3 : 4$. Whence it comes to pass, that Water appears not to be so deep, as it really is, by a fourth Part. But if BDA be Glass, then $CB : EB :: 2 : 3$.

Schol. From this Theorem is deduced the Method of finding the Focus C of Rays refracted by a plain Surface, *viz.* by drawing EB perpendicular to the refracting Surface DD, and therein taking C so, as that $CB : EB :: 1 : R$.

THEOREM III.

If the Surface BD of the Medium of *Fig. 32*, different Thickness be spherical, and the *33.* Object E be very far distant, then its Image will, to the Eye plac'd in AB the
 (1) Axis

Fig. 34,
35, 36,
37.

Axis of Radiation, and parallel to the incident Rays, (as ED,) appear in that Place C of the Axis AB produced as there is Occasion, where $BC : CA :: I : R$. But if the Object E be not very far distant, then its Image will, to the Eye placed in AB produced if there be Occasion, appear in that Place C of AB, where the Ratio compounded of the Ratio's of EA to AC and of CB to BE is equal to the Ratio of I to R. (*)

Demonst. The first Part of this Theorem (as it relates to Objects at so great distance from the refracting Surface BD, as that the Rays proceeding from each single Point therein, may, consider'd by themselves, be esteem'd parallel one to the other, so it) is thus demonstrated. Let ED denote any Ray falling on BD, and draw AD and CD, and produce them. The Sine of the Angle BAD or its Equal EDO is to the Sine of the Angle CDA (or its Complement to two Right) as CD to AC, or (by reason of the Smalness of the Arch BD) as CB to AC, that is, by

(*) I have comprehended all here relating to refracting Spherical Surfaces under this one Theorem for the like Reason, as I comprehended all relating to reflecting spherical Surfaces under one Theorem; and also that the Harmony between reflected and refracted Vision may the better appear.

Construction, as I to R. But EDO is the Angle of Incidence of the Ray ED, therefore DC is its refracted Ray. In like manner may it be shewn, that any other of the parallel Rays proceeding from (any Point of) the Object E after Refraction will pass through C. Therefore C will be the Focus of all such Rays, or which is the same, at C will appear in this Case the Image of an Object very far distant. Q. E. D.

The latter Part of this Theorem is thus demonstrated. Let ED be the incident Ray; draw AD and DC, and through C draw CH parallel to AD, and meeting ED in H. By reason of the suppos'd Smallness of the Arch BD, we may look on $CD = CB$, and $ED = EB$. Wherefore the Ratio of I to R is equal to the (Ratio of EA to AC and the Ratio of CD and ED together; that is, by reason of AD being parallel to CH, is equal to the Ratio of CD to ED and the Ratio of ED to DH together, or which is the same, to the) Ratio of CD to DH. But CD is to DH, as the Sine of the Angle (DHC, or its Complement to two right, viz. $ADH =$) EDO is to the Sine of the Angle (DCH $=$) ADC, or in some Cases to the Sine of the Complement of ADC to two Right. Wherefore EDO being the Angle of Incidence of the Ray ED, it follows that I

(I 2)

is

is to R, as the Sine of the Angle of Incidence of ED to the Sine of the Angle ADC. Wherefore (by *Theorem I*, of this Chapter) DC is the refracted Ray of the incident Ray ED. And ED being taken at pleasure, it is evident that C will be the Focus of all Rays proceeding from E after their aforesaid Refraction; and consequently at C will appear the Image in such Cases. Q. E. D.

From the first Part of this Theorem and the Corollary of *Theorem I*, arise several Corollaries; among which we shall here take notice of these following, viz.

Fig. 32. *Corol. 1.* Rays ED in the Air parallel to AB the Axis, after their Refraction at the spherical convex Surface DB of Glass, do converge to that Point, C, whose Distance BC from the Vertex B is equal to $3AB$, three Semidiameters of the Sphere.

Corol. 2. Rays ED in Glass parallel to the Axis, after their Refraction at the spherical convex Surface of the Air, do diverge from C that Point, whose Distance BC from the Vertex, is equal to $(3AB \text{ or})$ the Diameter of the Sphere: as *Fig. 33.*

Corol. 3. Rays eD in the Air parallel to the Axis, after Refraction at DB the concave Surface of the Glass, do diverge from C a Point three Semidiameters

meters $3AB$ distant from the Vertex : as
Fig. 32.

Corol. 4. Rays eD in Glafs parallel to the Axis, after Refraction at a spherical concave Surface of Air, do converge to C a Point distant from the Vertex B the Diameter of the Sphere (*): that is, $BC = 2AB$, as *Fig. 33.*

Corol. 5. In all the four Cases, which are the Converse of those mention'd in these four Corollaries, the Rays after Refraction will become parallel to (the Axis, *i. e.* to) a right Line drawn through the radiant Point and the Center of the Sphere.

As to the latter Part of this Theorem, there are eight principal Cases relating thereto, which may all be demonstrated after the same Manner, as those four of them, which are here mention'd, as being sufficient to our Purpose ; and are express'd by *Fig. 34, 35, 36, 37.* Of which *Fig. 34* and *35* relate to Rays proceeding out of a thinner into a thicker Medium, the other two, *Fig. 36* and *37*, to Rays proceeding out of a thicker Medium into a thinner. And in both Cases the former Figure relates to concave Refrings, the latter to convex. And in *Fig. 36*, is represented the Case, wherein the incident Ray ED is not refracted, but reflected into DC , according to what

was observ'd above, in *Schol.* 1 and 2 of *Theorem* I, of this Chapter.

Schol. 1. From the former Part of this Theorem is deducible the Way, how to find the Focus of parallel Rays falling on spherical Glasses, *viz.* by taking C in AB so, as that $BC : CA :: 1 : R$. And from the latter Part of this Theorem is deduced the Way, how to find the Focus of diverging Rays falling upon a spherical Glass, *viz.* by taking C in AB so, as that the Ratio compounded of the Ratio's of EA to AC, and CB to BE may be equal to the Ratio of I to R.

Schol. 2. If the Mediums propos'd be Air and Glass, and the Ray falls out of the Air upon the Glass (as *Fig.* 34 and 35) then the Focus C may be very easily found, namely, by making $\frac{1}{3}EA : AB :: EC : BC$. For by tripling the Antecedents, $EA : (AB =) AD :: 3EC : (BC =) DC$. But EA is to AD, as the Sine of the Angle EDO is to the Sine of the Angle DEA; and $3EC$ is to DC, as the Triple of the Sine of the Angle EDC or HDC is to the Sine of the Angle DEA. Wherefore the Sine of the Angle EDO is the Triple of the Sine of the Angle HDC, and consequently is to the Sine of the Angle ADC, as 3 to 2. Therefore DC, or De lying in the same direct Position, is the Refract of the Ray ED falling out of the Air upon Glass.

When

When the Ray passes from Glass into the Air (as *Fig. 36* and *37.*) then the Focus may be found, according to the same Principles, by taking C so, as that $\frac{1}{2}EA : AB :: EC : BC.$

Schol. 3. By what has been said under the foregoing Theorems, there being given one Focus of a given Lens, the other may be found. Where it is to be noted, that by a *Lens* is denoted a (diaphanous or) transparent Body (such as Spectacles, Burning-glasses, &c.) of a different Thickness from the ambient Transparent, terminated by two Surfaces, which are either both spherical, or the one plain and the other spherical. A right Line perpendicular to both its Surfaces, is call'd the *Axis* of the Lens. The Points, where the Axis intersects the Surfaces, are call'd the *Vertex's*; one the Vertex of *Incidence* or *Immersion*, the other of *Emersion*, accordingly as it is in that Surface, on which the Rays first fall, or through which they pass out again. The *Thickness* of the Lens, is the Distance between its Vertex's. To which add, that where more than one Lens is us'd, that which is next to the Object, is thence call'd the *Object-Lens*; and that which is next to the Eye, is thence distinguish'd by the Name of the *Eye-Lens*.

Fig. 36.

Fig. 38.

Now if the Focus before Incidence or Immersion be given, and the Focus after Immersion be sought; let there be found first the Focus of the Rays after their Refraction at that Surface of the Lens, on which they first fall; and this is done by *Theorem II*, if the Surface of the Lens be plain; but if it be spherical by *Theorem III*, namely, by the former Part of it, if the Rays are parallel; and by the latter Part, if diverging or (*) converging. Having thus found the Focus of the Rays after Refraction at the first Surface of the Lens, (that is, while they pass through the Lens, whence the said Focus is call'd the *Focus of Transition*.) in like Manner may be found their Focus after Refraction at the second Surface of the Lens, or rather at that Surface of the ambient Medium, which is contiguous to this second Surface; that is, the Focus after Emerision.

If there be more than one Lens propos'd, after the like Manner is the Operation to be carried on in respect to each.

(*) Namely, according to what has been observ'd of converging Rays in *Schol 3. of Theorem I. of this Chapter.*

In like Manner, there being given the Focus of one given Lens or more, thereby may be found the Focus before Incidence, that is, the optical Instrument being given, thereby may be determin'd the Distance of the visible Object.

THEOREM IV.

The Image CT of a radiant plain Surface EF, made by a refracting Surface BD, whether plain or spherical, will be also a plain Surface. Fig. 39.

Demonst. Let A be the Center of BD the refracting Surface, and AE be perpendicular to EF, meeting with BD in B, Let C be the Focus of the Rays diverging from E and refracted by BD. From F, any Point taken at Pleasure in the radiant Plain draw, FA meeting with BD in D, and the Plain CT in T. The Ray DC will be the Refract of ED; and let BH be the Refract of FB. Because the Angle EAF is suppos'd to be small, therefore the Arch BD may be look'd on as a short right Line; and a Circle drawn with the Diameter BF will pass through the Points D and E, because of the right Angles BEF and BDF. Whence the Angles EBF and EDF, (*viz.* the Angles of Incidence of the Rays FB and ED) being in the same Segment, are equal; and consequently

sequently their refracted Angles ABH and ADC will be equal. Therefore by reason of the equal Angles at A, the Triangles BAH and DAC are equiangular, and BA is to AH as DA to AC; and, whereas $BA=DA$, therefore $AH=AC$. But because the Angle EAF or TAC is very small, therefore (in a manner) $AT=AC$; and therefore AH may be look'd on as equal to AC, that is, the Focus of the radiant Point F, is placed as near as may be in the Plane CT. And the Point F being taken at pleasure, it holds good also in all other Points of the Plane EF, that their Focus's, are in the Plane CT; that is, that the Image of the radiant Plane EF will in this Case be also a plain Surface. Q. E. D.

Corol. 1. Hence it follows, that the Image of the radiant Plane EF, whereto the Axis of the Lens is perpendicular, will also be a Plane parallel to the former. For the Image CT is to be conceived as the plain Surface, which sends forth Rays upon the second Surface of the Lens.

Corol. 2. If the Angle EAF be too great, so that AT is so much longer than AC, as not to be capable of being esteem'd equal thereto, then the Image of the Plane EF will be concave towards A, if BD be the convex refracting Surface of a thick-

a thicker Medium, or the concave refracting Surface of a thinner Medium ; and on the contrary.

THEOREM V.

The Angle fGb , under which the Image fCb appears from G the Vertex of Emerfion, in a Lens, is equal to the Angle FBH under which the Object appears from B the Vertex of Incidence in a Lens. Fig. 38.

Demonst. Let GB be the Axis of the Lens, and perpendicular to the visible Plane or Object FEH at E , and consequently (by *Corol. 1, Theorem IV.*) perpendicular to its Image fCb . Draw BF , BH , Gf , and Gh . Of the innumerable Rays flowing from the Point F , and after Refraction at the Lens again united at f the Image of the Point F , let two be chosen, whereof one FB meets the Lens at B its Vertex of Incidence, and being there refracted tends to p the Focus of Transition in respect of F ; and being again refracted at L tends towards f . The other Ray FD being first refracted at D , tends directly to p , till passing out of the Lens at G the Vertex of Emerfion, it is there refracted again, and thence proceeds directly to f . The Angle of Incidence of the Ray DG is DGB , and its Refract is CGf ; and the Angle of Incidence

dence of the Ray LB, which would (by the *Corollary* of *Theorem* I,) be refracted into BF, is the Angle LBG, and its Refract EBF. Now because the right Lines pB and pG are to be esteem'd equal, (by reason that the Thickness of the Lens is not to be consider'd,) therefore the Sines of the Angles DBG and LBG being proportional to the said Lines pB and pG , and consequently the Angles themselves DGB and LBG, and therefore their refracted Angles CGf and EBF will be equal. In like manner it may be prov'd, that the Angles CGb and EBH are equal. Therefore the Angles FBH and fGb are equal. (And the like may be shewn in any other Case,) Q. E. D.

Corol. Hence it follows, that a radiant Line is to its Image made by a Lens, as the Distance of That from the Vertex of Incidence is to the Distance of This from the Vertex of Emerfion, or (the Thickness of the Lens being not regarded) as their Distances from the Lens. But if the Radiant be a Surface (the homologous Lines will keep in the same Ratio, but) the said Surface will be to its Image (in the said Ratio duplicated, or) as the Squares of their Distances from the Lens. Hence may be determin'd the Ratio, which the last Image, (*i. e.* that which is immediately seen by the Eye) made by the Means of

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of one Lens or more, has to the visible Object, whose Image it is.

Schol. From what has been said, may be accounted for and illustrated the Affections of (or several Particulars relating to) refracting Surfaces, whether plain or spherical. Among which Affections the more remarkable are these that follow, *viz.*

1. If the prime Radiant or Object be far distant, and the Surface PCQ of the Lens be plain, the Surface PDQ be of Glass and convex, and the ambient Body be the Air, then (by *Corol. 4, Theorem III*) D*b* will be equal to the Diameter of the Sphere PDQ; (for by *Axiom 1*, the Rays parallel to BC pass through the plain Surface PCQ, whereon they fall perpendicularly, without being refracted :) and consequently by *Theorem IV* and V, *f*be will be the Image of the far distant Object, from which proceed the Rays FBE; and therefore the said Image will appear in an inverted Situation with respect to its Object: Namely, the Point F of the Object, and the correspondent Point *f* of the Image, will be on different Sides of B*p* the Axis; and so of the other Points B and *b*, E and *e*. And therefore, if F be the lower Part of the Object, *f* will be the upper Part of the Image; and so as to the rest agreeably.

2. But

Fig. 40.

Fig. 41.

2. But if the Surface PCQ be convex, and PDQ plain, then (by *Corol. 1, Theorem II*, and *Corol. 1, Theorem III* of Dioptricks) Cb will be equal to the Diameter of the Sphere PCQ, and a third Part of the Thickness of the Lens over. Or neglecting the Thickness of the Lens, (as is usual in the Object Lens of a Telescope,) in this as well as the former Case, and consequently in general of any plain convex Lens, it may be said, that the Image of an Object far distant, made by such a Lens, will be equal to the Diameter of the Sphere.

Fig. 42.

3. If the Lens PQ be of Glass, and equally convex on both Sides, then (by *Corol. 1. Theorem III*, and *Theorem IV* of Dioptricks) Db will be equal to the Semidiameter of either Sphere. Lastly, in

Fig. 43.

a whole Glass Sphere, the Image of an Object far distant will appear at the Distance of half the Semidiameter from the Sphere: For in this Case Regard must be had to the Thickness of the Lens.

Of Burning-glasses.

4. In all these or the like Cases, if the Sun be the far distant Radiant or Object, and the Lens be considerably larger than the Sun's Image, in the Place of the Image will be the *Burning point*: And any such convex Lens will burn stronger than a concave Reflector (*ceteris paribus*)

bus) by reason more Rays are lost by the latter than by the former.

5. If in the foremention'd Place of the Image there be placed a lucid Body it self; then the Image of the said Body will appear at a great Distance; that is, the said lucid or shining Body will cast a Light at a great Distance, so as that Things so far distant may be clearly seen by the said Light. And this is the Manner, whereby a Candle, put into a Dark-Lantern with a convex Chrystal, gives a Light at so great a Distance as usual

Of Dark-
Lanterns
with Con-
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ses.

6. If the Radiant or Object be not (as afore) far distant, but yet is more distant from the Lens than is the Place where would appear the Image of a far distant Object; besides the Affection aforemention'd, (*viz.* that the Image will appear inverted) there will be in this Case also these other Affections. Namely, as the Radiant comes to the Lens, the Image will go from it, and contrariwise; 'till the Radiant being come into the Place, where would appear the Image of a far distant Radiant, its own Image becomes far distant. All which Affections may be observ'd in a Dark-Chamber, into which Objects at several Distances send their Rays through a convex Lens or Glass. The Place of the Image of each Radiant Object may be distinctly known, by observing

serving where the Image of the said Object does appear most distinctly on a Sheet of White Paper or the like.

Of a Magick-Lantern.

7. And on these same Principles depend the Mechanism of (what is call'd) a *Magick-Lantern*; namely, whereby the Images of little Pictures are represented on a white Wall or the like.

8. If the Radiant be nearer to a convex Lens, than the Place where would appear the Image of a far distant Radiant or Object, then its Image will appear on the same Side of the Lens with the Object it self; and its Place may be determin'd from the given Place of the Radiant, as afore. The Image in this Case will always appear in the like Situation with the Object, and greater than it; and if the Object comes to the Lens, the Image will also; if that goes from the Lens, this will also; only the Image will in both Cases move quicker.

Fig. 44,
& 45.

9. If the Lens be concave, then (by the respective Corollaries of *Theorem III* of *Dioptricks*) the Image self made by a plain concave Lens, PQ, will be distant the Diameter of the Sphere. PDQ, and will be erect on in the like Position with the Object, and on the same Side with the Object reckoning from the Vertex D. If the surface PCQ be also concave, and the Object not far distant, its Image may

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may be determin'd after the like Method, by finding (according to *Theorem* IV of Dioptricks) the Image of that Point of the Object which is situated in the Axis of the Lens. In respect to which Image, besides what has been already said, it is to be observ'd, that it will together with the Object come to or from the Lens, but slower.

10. Further, it is here observable, that on the foremention'd Theorems depends the Mechanism or Contrivance of the several optick Instruments, made use of to help the Eye, or to supply the several Defects of our Sight. For, as the Eye is a natural Organ or Instrument, design'd to this End, namely, that on the Bottom thereof may be painted the distant Images of outward Objects; so it is manifest that one and the same Eye can see distinctly only at a certain Distance from the visible Object. Could the Eye always place it self at such a Distance from the Object, as is necessary to distinct Vision, then (supposing the other Requisites to Sight to be given, such as a due Degree of Light, &c.) the Eye might always see distinctly. 'Tis true indeed that the Distance requisite to distinct Vision is not rigidly determin'd to one single Point, but admits of some Latitude; forasmuch as the same Eye can so alter its Figure according to

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the various Distance given, that it may be look'd on not as one Eye but several. But then, since this Latitude of Distance is contain'd within certain Bounds, beyond which the Eye can by no natural Means see distinctly, hence arises the Need of artificial Instruments. And any Lens, or spherical reflecting Glass, is sufficient to remedy this; forasmuch as thereby the Image of such an Object, as we can't come to as we would, may (by Help of the Theorems above demonstrated) be brought near us, so as that we may have a distinct View thereof. And of this Kind are those single Glasses, us'd by such as can't see well at a common Distance, and apply'd by them to one of their Eyes, when they would discern the Face or Picture of a Person (or the like) that is farther distant from them than they can distinctly see with the bare Eye. As the Word *Telescope* does in the Greek Tongue denote an Instrument which helps us to *see far off*, so these single Glasses may be call'd *single Telescopes*, to distinguish them from such Telescopes as are made of two or more Glasses.

II. But because (beside distinct Vision) it is often requisite in humane Affairs, that the more minute Parts of an Object should be discern'd; and because it is found by Experience, that an Object
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which appears under a less Angle than of one Minute, appears but as a Point, or so as that its Parts can't be distinguish'd one from the other : Hence it often comes to pass, that the Object being brought so near to the Eye, as that its Particles to be examin'd may be seen under a sensible Angle, thereby the Object it self becomes too near to the Eye, so as not to be within the Bounds requisite to distinct Vision. This Inconveniency, when alone, may also be remedied by any given Lens or reflecting Glass ; forasmuch as the Image of the said Object (*) may by the said Lens or Glass be represented (in any given Measure, *i. e.*) so as to have any given Ratio or Proportion to its Object.

(*) How this may be done in respect of a reflecting Glass, is shewn above in *Schol 7, to Theorem V, or last of Catoptricks*. How the same may be done in respect of a Lens or refracting Glass, shall be here shewn. Let a Glass Lens be given (*Fig. 46*) the Semidiameter of whose foremost Surface is AB, the Semidiameter of the hindermost Surface CB. Draw CM as you please ; and let the Ratio assign'd be such as ought to be between the homologous Lines of the Radiant and its Image, and be denoted by the Ratio of MD to DC. Draw AM, whereto through B draw the Parallel BE, meeting CM in E ; ME doubled will be the sought Distance of the Radiant from the Lens. And after the same Manner may be found, in respect of any given Lens, the Position, or Place where any Radiant is to be put, that (its Image made by the Lens may be equal to a given Figure, which is similar or like to the Radiant, *i. e.* in short, that) the Radiant may have a given Ratio [to its Image made by the Lens.

Fig. 46.

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And of this Kind are those single Glasses us'd to see the Shape or Parts of small Insects, and the like, and thence call'd *Microscopes*, the Word importing in the Greek Language an Instrument, whereby we see *small* Things. To these may be referr'd Spectacles, of which more by and by.

But if both the foremention'd Inconveniencies concur, then they are not to be remov'd but by the joynt Help, either of a Lens and Reflector together, or else of more than one Lens, or lastly of more than one Reflector. And how Instruments consisting of two or more of these are made, shall be next illustrated.

Fig. 46.

13. Let R denote the Object given, S the given Angle under which the Object is to be represented, D the given Distance of the Sight, and L the given Distance of the Eye from the Lens. Make the Triangle AOB, so as that the vertical Angle at O may be equal to the given Angle S, and OE the Perpendicular from the Vertex to the Basis may be equal to the given Line D. If you'd have the middle Point of the Object be in the Axis of the Lens or Reflector, (which is most convenient for Practice) the Triangle AOB must be made an Isosceles. Take $OV=L$, and at V place a Lens or Re-

Reflector so, as that its Axis may fall in with OE.

Then having E thus given for one Focus of the Lens or Reflector, by the respective *Theorem III* of Catoptricks or Dioptricks or their Scholium, find the other Focus e ; namely, that so taking e for the Focus before Incidence, E may be the Focus after Inflection at the Lens or Reflector. Through e draw aeb parallel to AEB, meeting with Va and Vb (drawn through V, and parallel to VA and VB) in a and b .

It follows from *Theorem V*, of Catoptricks or Dioptricks (respectively, as you calculate for a reflecting or refracting Glass,) that if aeb be the Radiant, AEB will be its Image. Wherefore if the Object given R, (which is here suppos'd to be near at hand) and a second Lens or Reflector be so plac'd, according to *Schol. 7*, *Theorem V* of Catoptricks, or according to the Note before added to *Schol. II* of this *Theorem V* of Dioptricks,) that the Image of the Object made by this second Lens or Reflector, may appear just in the Place, Situation, and Bigness of aeb ; there will be thus made such a Microscope as is requir'd. For aeb will be the Image of the given Object R, the Image of which Image, and that seen by the Eye, will be AEB. And this will appear

under the Angle AOB equal to the given Angle S ; and in the Distance OE equal to the given Distance D, and consequently will appear distinct ; and lastly, the Distance of the Eye from the Lens or Reflector next to it in V, will be equal to L, the given Distance in this respect. And so the Instrument will be in all Respects such as is requir'd.

14. If the Instrument be made for the Eye of an aged Person, then the right Line OE becomes infinite, and consequently the Point *e* is to be found by *Theorem III* of Catoptricks and Dioptricks. As for the rest of the Work, it is the same as afore.

If the Object propos'd to be distinctly seen be far distant, then having all (as afore) given, but the Angle S, a Telescope may be made fit for the Purpose thus. In a right Line extended toward the remote Object propos'd, take from O forward OE and OV equal to the given D and L ; and in V place a Reflector or Lens having its Axis in VE. The Focus E of the Lens or Reflector plac'd at V being thus given, find the other Focus *e*, namely, that this being taken for the Focus before Inflection, E may be the Focus after Inflection. In the right Line OE so place a second Lens or Reflector, having also its Axis in OE, that the I-
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mage of the propos'd Object made thereby, may appear in the right Line *aeb* perpendicular to *OF* ; and so there will be made such a Telescope as is requir'd. For the first Image of the far distant Object propos'd will be in *aeb* ; and the Image of this Image will be in *AEB*, the Distance of which *AEB* from *O* is by Construction equal to the given *D*, and consequently distinct to the Eye ; and also the Distance of the Eye from *V* is equal to the given *L*. And from hence may be found the Angle, under which the last Image is seen.

16. By the Help of three or more Lens's or Reflecters, an Instrument may be made to represent even a far distant Object under a given Angle. Namely, let any one Lens or Reflector form the Image of the Object propos'd ; and then with the other two make (as afore) a Microscope, representing the first Image (as being a given Radiant nigh at hand) according to the Conditions propos'd ; and so there will be made such a Telescope as is requir'd. In like Manner, a Microscope being made of three Lens's or Reflecters, by adding a fourth it will become a Telescope. In all which Cases is known the Ratio or Proportion, which the Image seen by the Eye has to the Object, or (which comes to the same) the Proporti-

on, which the Angle under which the said Image is seen, has to the Angle under which the Object is seen without the Instrument, (which, in respect to far distant Objects, is as the Distances of the Lens's or Reflecters from the common Focus *e.*) And consequently from hence (according to the Corollary of *Theorem IV, Chap. III and IV,*) may be computed how much the Instrument does help the Sight.

Fig. 48,
& 49.

17. I shall next adjoin some Observations in reference to a dark Chamber, as was abovemention'd in *Schol. 2, Theorem IV, Chap. II.* Namely, it was there observ'd, that Rays let into a dark Chamber through a naked little Hole, will paint the Images of their respective Radiants on a Sheet of white Paper held against the Hole. To which it is here to be added, that the same will be done thro' a much larger Hole, if a Lens or spherical Glass be placed therein, with this Difference, that the Images of outward Objects will be painted much more lively and distinctly this Way than the other without a Lens; which is thus accounted for, Through the little Hole (*Fig. 48*) must necessarily pass fewer Rays from the outward Objects than through a much larger Hole (*Fig. 49.*) And not only so, but those fewer Rays, which pass through

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through the bare Hole are dispers'd ; whereas by a convex Lens or Glass all the Rays proceeding from any one Point of the Object, are united again at a certain Distance, and so render the Image much more lively and distinct, than the other Way. This is illustrated *Fig. 48* and *49*. Whence it is also obvious, why in both Cases, the farther the Paper is held from the Hole, the bigger the Image will appear ; as also why the farther the outward Objects are from the Hole, the less their Images will be ; and why the Image made by a Glass will appear most distinctly at the Distance *ac* (*Fig. 49*) with other such like Particulars.

18. But it is observable, that in both Cases, either with or without a Glass, the Image will appear inverted in respect of the Object, as has been afore observed under *Theorem IV, Chap. II*, and is evident from *Fig. 48* and *49*. And from *Fig. 48* it is evident, that when the Image *abγ* is painted erect on the Bottom of the Eye, then the immediate Radiant *abc* it self is seen inverted by us ; and on the contrary, when the Image *abc* is painted inverted on the Bottom of the Eye, then we see the immediate Radiant *ABC* erect. For as in the said Figure the second Image *abγ* is erect in the Eye, so if the Eye were put in the Place of the Paper,

per, the first Image *abc* would be painted therein inverted, in respect of the Object ABC.

19. This being so, the Manner how that Sensation, which we call Vision, is perform'd, seems to be the same, as that wherby a blind Man by the Thrust of the End of several Sticks against his Hand, can thereby distinguish, which Stick thrusts from above, which from below. Namely, so by the Sensation caus'd by the Ends *a* and *b* of the Rays *Aa* and *Bb* and *Cc*, touching the Retina, we are naturally endued with a Faculty of readily distinguishing, from whence each Ray proceeds, *viz.* that the Ray *Aa*, as touching the lower Part of the Retina, does proceed from the upper Point *A* of the Object; and the Ray *Cc* as touching *c* the upper Point of the Retina, does proceed from the lower Point *c* of the Object; and the Ray *Bb* as touching *b*, the Middle of the Retina, does proceed from *B* the Middle of the Object. And consequently we are enabled to judge of the Situation of Things aright, by receiving their Rays after such a Manner as (it appears from the foresaid Experiments) we do: Which is the same as to say, that we see Things in their Erect or due Situation, by having their Images painted on the Retina in an inverted Manner; and

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and contrariwise, we see Things inverted, by having their Images painted on the Retina in an erect Manner,

20. I shall now, in the *last* Place, explain the Mechanism of *Spectacles*, or how they come so to help the Eye. It is then evident from the Rules of Dioptricks, that convex Glasses turn the Rays inward, or towards the Axis of the Glass; and the more convex they are, so much the more inward the Ray is turn'd. It is also found by Observation or Experiments, that the Chrystalline Humour has the same Effect as a convex Glass. Wherefore, when the Surface or Coat of the Chrystalline Humour is of such a due Convexity, as that it makes the Rays of the several Points of an Object to unite again in so many several Points just upon the Retina, then Vision is duly perform'd, as *Fig. 50.*

of Spectacles.

Fig. 56.

21. But when the Chrystalline Humour is not convex enough, but too flat, (as it grows through Age,) then the Rays passing through it are not united soon enough, *i. e.* are not united at the Retina, but their Point of Union is further behind the Retina, and consequently then the Eye can't see duly or distinctly, but only confusedly, the Objects, from whence proceed the Rays. Wherefore to help this, convex Glasses, (call'd Spectacles)

Fig. 51.

cles) are us'd, whereby the Rays are made to unite in their proper Place, or at the Retina. In order thereto, the more flat the Chrystalline Humour is, that is, the more aged the Person is, the more convex must the Spectacles be.

Fig. 52.

22. On the other Hand, if the Surface of the Chaystalline Humour be too convex, (as is the Case of short sighted Persons,) then the Rays are thereby united too soon, or before they come to the Retina, and so Vision is not duly perform'd. Hence such Persons hold Things very near to their Eyes, to see them the better, because (according to the Rules of Dioptricks) the nearer the Object is to the (Lens, or in this Case) the Chrystalline Humour, the further will be the Union of its Rays on the other Side. Hence also concave Spectacles helps such Persons Sight, by making the Rays unite in their proper Place, *viz.* at the Retina. Hence, lastly, such Persons see better and better, as they grow elder; because the Chrystalline Humour grows more and more flat, or less convex; so as to come but to a due Convexity in their old Age: Which by the way is the Reason, that such People, as are short-sighted when Young, want not Spectacles when they are Old. And this is sufficient to our Purpose concerning Dioptricks.

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Of Perspective.

IN Perspective the chief Things (beside the Eye and the Object) to be consider'd, and therefore explain'd, are those which are contain'd in the following Definitions.

I.
Definitions
of Terms
us'd in
Perspective

The (†) *Geometrical Plane* is a plain Surface parallel to the Horizon, and placed lower than the Eye, in which we imagine the visible Objects without any Change, (except it be reducing a great Figure into a small one,) and upon which the Situation of the Object to be represented in Perspective is describ'd.

2.
The Geometrical
Plane,
what.

The *Situation* (call'd also the *Ichnography* or *Plan*) of an Object is its Orthographical Projection upon the

3.
The Situation of an
Object,
what.

(†) It is also called the *Basis*. This and the following Planes, &c. here denoted, are not represented in Figures or Cuts, they being not easie to be so apprehended by young Students; but they may be more easily apprehended by the bare Imagination, or else by actually placing so many Plains in such a Manner.

(*) geo-

(*) geometrical Plane, or, that Figure which is describ'd upon the Geometrical Plane, when from all the Points of the Object right Lines are drawn perpendicular to the said Geometrical Plane. Thus the (Plan or) Situation of a right Cylinder is a Circle, and the Situation of a right Cube is a Square.

4.
The Picture, *what.*

The *Picture* is a plain Surface, suppos'd as transparent as Glass, and always plac'd at some Distance between the Eye and the Object, to represent in it the Object in Perspective; whence the Picture is also call'd the *Perspective Plane*.

5.
The Ground-line, *what.*

The Picture stands (usually †) perpendicular) upon the geometrical Plane. Whence the common Section of the Picture and the geometrical Plane, is call'd *the Base of the Picture*, or the *Base-line*, and also the *Ground-line*.

(*) If the Orthographical Projection be (not upon the geometrical Plane, but) upon a Plane parallel to the Picture, then it is call'd *Front*; and if it be upon a Plane parallel to the vertical Plane (describ'd §. 11) it is term'd *Profil*.

(†) Sometimes the Picture is inclin'd to the geometrical Plane or Horizon, and has also a curve Surface, as when the Surface of an arch'd Roof is to be drawn. But this being not common, the Picture is look'd upon throughout this Chapter as a Plane perpendicular to the geometrical Plane or Horizon.

The

The *Horizontal Plane* is a plain Surface parallel to the Horizon, and which is conceiv'd to pass through the Eye, and to be perpendicular to the Plane of the Picture.

6.
The Horizontal Plane, what.

The *Horizontal Line* is that right Line, in which the horizontal Plane and the Plane of the Picture intersect, and which therefore is parallel to the Ground-line.

7.
The Horizontal Line, what.

The *Principal Ray* is a right Line drawn from the Eye, perpendicular to the Plane of the Picture, which therefore must run along the horizontal Plane.

8.
The Principal Ray, what.

The *Point of Sight*, call'd also the *Principal Point*, is the Point where the Picture is cut by the principal Ray, which must necessarily be in the horizontal Line.

9.
The Point of Sight, what.

The *Point of Distance* is a Point in the horizontal Line, whose Distance from the Point of Sight, is equal to the Length of the principal Ray.

10.
The Point of Distance, what.

The *Vertical Plane* is a plain Surface, which going along the principal Ray, is perpendicular to the Horizon, and so to the geometrical Plane, and also to the Picture: The common Section of the Picture and vertical Plane being call'd the *Vertical Line*, and being also the Measure of the Height of the Eye; and the common Section of the geometrical and vertical

11.
The Vertical Plane and Line, &c. what.

vertical Planes being call'd the *Line of Station*.

12.
*The Appearance
of a Point,
what.*

The *Representation* or *Appearance* of any Point of an Object is that Point, where the Picture is cut by a right Line drawn from the Eye to the Point propos'd of the Object.

13.
*The Accidental
Point,
what.*

The *Accidental Point* of a right Line is the Point, where the Picture is cut by a right Line drawn from the Eye parallel to the Line propos'd. Whence it is easie to conclude, that all Lines, which are parallel to the Picture, have no accidental Point; and that all other Lines, which are parallel one to another, have the same accidental Point. It is also observable, that all right Lines, which are perpendicular to the Picture, have the principal Point for their accidental Point; and that all those, which make an half-right Angle with the Picture, have one of the Points of Distance for their accidental Point.

14.
*Observa-
bles from
what has
been said.*

It is also observable from what has been said, that all the Parts of an Object, which are lower than the Eye or horizontal Plane, ought to be represented in the Picture below the horizontal Line. And on the contrary, all above the horizontal Plane, or higher than the Eye, ought to be represented in the Picture above the said horizontal Line. In like Manner, all those

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hose Objects, which with respect to the Eye are on the right Hand of the vertical Plane must in the Picture be represented on the right Side of the vertical Line; and such as are on the Left, must be represented on the left Side of the said vertical Line.

In order to perform any Operation, the Sheet of Paper, which you would work upon, must be divided into two Parts, by the Line AB (*Fig. 53*) which will represent the Ground-line. The upper Part of the Paper, *viz.* ABDD, must be taken for the Plane of the Picture; and the lower Part ABYZ of this Paper for the geometrical Plane.

15.

How to describe on Paper the Plane of the Picture, and the Geometrical Plane.
Fig. 53.

The next Thing is to know, (under what Angle the Picture ought to be seen, that whatever is represented therein may appear in just Proportion, and may easily be taken in by the Eye at one View; that is in short,) how far the Points of Distance ought to be from the Point of Sight. Let there be propos'd a Square, which takes in all that you would represent in Perspective. And let the Ground-line AB be of equal Length with one Side of the Square propos'd. Wherefore, if the Eye of the Spectator was at G, he could not see the two Ends A and B, because the Eye could see, (†) at most. but

16.

How to find or set off the Points of Distance.

(†) According to *Theorem II.* of *Chap. II.*

so much as is comprehended under the right Angle GHI. But if the Eye were at K, it could see the Ends A and B, because the Angle AKB is less than a Right, viz. 80 Degrees. And the Eye would see the Ends A and B better at L, and better yet at M, where the Angle AMB is 60 Degrees. And from such a Distance the Objects (to be represented in Perspective, and contain'd in the foremention'd Square) may be seen in Perfection, the visual Angle AMB being neither too great nor too little. The Angle ANB would also be very well. Wherefore having bisected AB in F, and from F drawn the Line FV perpendicular to DD for the vertical Line, and consequently V being the visual Point or Point of Sight, from the said Point V set off the Distance ((*) FK or rather FL, or) FM, or FN on each Side upon the horizontal Line DD, viz. to D and D, which will be the Points of Distance requir'd or sought.

17.

A general
Rule con-
cerning the
same.

And this may be look'd on as a general Maxim, viz. that the Distance betwixt the Point of Sight V and the Points

(*) If the Distance VD were less than FK, then the Perspective Square ABCE (concerning which see § 17) would appear irregular, because it would be seen under an oblique, and so too great an Angle.

of Distance D and D must be (at least) equal to FA or FB. And it will not be amiss to let this Distance be somewhat greater. And, if in this Case the Plane of the Picture be not wide enough to receive two Points of Distance, there must only one be mark'd; and the principal Point or Point of Sight V may be plac'd on one Side, as *Fig. 59, &c.*

The Points of Distance D and D (*Fig. 53*) being found, draw the Lines AV and BV, as also AD and BD. And the Figure ABCE will be the Appearance (in the Picture) of the Square, propos'd to be represented in Perspective; which Appearance of the said Square is in short call'd the *Perspective Square*.

18.
The Perspective Square, what.

Proceed we now to the Representation (in the Picture) of Points, Lines, and Planes. And first, how to find in the Picture the Representation or Appearance of a *Point* given in the geometrical Plane. From C the given Point (*Fig. 54*) let fall upon the Ground-line AB the Perpendicular CE; and from the Point E draw to the principal Point V the Line VE. Upon AB set off EF (or EA) = CE; and then draw FD (or AD either of) which will upon VE give at *c* the Appearance of the given Point C.

19.
How to find in the Picture the Appearance of a Point given in the geometrical Plane.
Fig. 54.

20. Secondly, how to find in the Picture the Appearance of a *right Line* given upon the geometrical Plane. Suppose MN (*Fig. 55 and 56*) to be the right Line given. Find (as afore) the Appearance of the Point M, *viz.* at *m*, and then the Appearance of the Point N of the given Line MN, *viz.* at *n*. Then join *m* and *n*, the Line *mn* will be in the Picture the Appearance of MN the Line proposed (*).

21. When the Line given on the geometrical Plane is (not a right Line, but) a curve Line, then from several Points of it draw Perpendiculars to the Ground-line; by means of which, and what has been said, may be found in the Picture the Appearances of all those several Points, which being join'd by a curve Line, that Curve will be the Appearance of the Line propos'd.

(*) That every Thing you design to represent in Perspective, may appear in just Proportion, it is requisite sometimes to have the Eye at a great Distance from the Picture, which may hinder from marking either Point of Distance upon the horizontal Line. In which Case you must mark only half of the Distance of the Eye from the Picture upon the horizontal Line, as (*Fig. 57*) from V to Q, which will represent one of the Points of Distance. And then you must only set off half the Perpendicular HI upon the Ground-line AB, from I to P. Then draw QP, and the Point *b* will be the Appearance of H the Point propos'd.

Thirdly,

Thirdly, To find in the Picture the Appearance of a plain Figure given on the geometrical Plane, suppose the Square CEGH *Fig. 58*. Find the Appearance (as has been afore shewn) of its four angular Points C, E, G and H; which will be *c, e, g* and *h*. Join the said (last) Points, and the Perspective Square *cegh* will be the Appearance of the given Square CEGH, when seen foreright, or when the Line KL, of the given Square falls in with VF the vertical Line. If the given Square be all on one Side of the vertical Line VF, (as *Fig. 59*) then the Perspective Square *cegh* (in *Fig. 59*) will be the Appearance of the given Square CEGH.

Where also it is to be noted, that the Appearance *g* of the Point G may be found also after this Method. Namely, in the Ground-line AB take the Point B at pleasure, and draw BV. Then set off BM equal to the Perpendicular IG, and draw MD, which upon the Ray VB will give the Point N. Through which NC being drawn parallel to AB, will upon the Ray VI give the Point requir'd *g*. And this Method may be used, when the Perpendicular IG is so long, that the Length thereof can't be set off upon the Ground-line AB w^lthin the Picture.

In the same Manner may be found in the Picture the Appearance of any other plain

22.

To find in the Picture the Appearance of a plain Figure given in the geometrical Plane.
Fig. 58, 59.

23.

A Remark.
Fig. 59.

24.

Another Remark.

plain Figure, *viz.* Triangle, Pentagon, Heptagon, &c. Namely, by distinctly finding the Appearance of each angular Point of the plain Figure propos'd, and then joining the Appearances so found by right Lines.

25.
To find in a
Picture the
Appearance
of a Circle
given in the
geometrical
Plane.
Fig. 60.

It shall here be only observ'd farther in reference to plain Figures, that to find in the Picture the Appearance of a Circle given in the geometrical Plane, there must be describ'd about it a Square, as (Fig. 60,) the Square EFGH about the Circle ILKM. One Side of the Square, *viz.* EF (or GH) must be parallel to the Ground-line AB, and consequently the other Side EH (or FG) must be perpendicular to AB. Draw the two Diagonals EG and FH, which intersecting at O the Center of the Circle, will give upon IKL, M the Circumference of the Circle, these four Points, P, Q, R and N. Then find in the Picture the Appearance of the Square EFGH, which will be the Perspective Square *efgh*, and also the Appearances of the Points L, Q, K, R, M, N, I and P; which will be in the Perspective Square the Points *l, q, k, r, m, n, i* and *p*. A Curve drawn through these last Points will determine the Appearance of IKLM the Circle propos'd. Where note, that the Appearance of the Circle will also be a true Circle, if the Circle pro-

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Propos'd be seen fore-right, that is, if its Line (*) LM, being perpendicular to AB, make one right Line with VF the vertical Line. But if the Circle propos'd IKLM be situated (as *Fig. 61*,) then its Appearance will be (as there) elliptical.

Proceed we now to shew, how to represent in Perspective a *Floor of equal Squares* seen fore-right, without a geometrical Plane. Divide the Ground-line AB (*Fig. 62*) into as many equal Parts as you please, each of which will repre-

26.

To represent a Floor of equal Squares seen fore-right. *Fig. 62.*

(*) To have the Appearance of a Circle in the Picture, to be likewise a Circle, the Distance between the Eye and the Picture must be of a determinate Length, which may be found thus. Having produc'd the Side of the circumscrib'd Square EFGH, 'till it meets at right Angles with the horizontal Line DD in a Point as S, draw SL, and cut off from it SC=SV, and the remaining Part LC (of SL) will be the Distance between the Eye and the Picture, or VD equal to the said Distance. If VD be given, and you would find VE (or the Distance between the horizontal Line DD and the Ground-line AB, that is) the Height of the Eye requisite, that the Appearance may be also a true Circle; the rectangular Triangle SVL shews, that the Square of the Line EL (or of the Ray of the given Circle IKLM), must be taken from the Square of the Line LS, and the Side ES of the remaining Square will be the Height requir'd. Note further, that the Perspective Square *efgh* being found, the Mid-point X of the Line *mL* (or *ml*) will be the Center of the Perspective Circle; upon which therefore Center the said Circle may be drawn without any more ado, or without finding the Points *l, q, k*, &c. Lastly, note, that in *Fig. 60*, the Perpendiculars EI and TN are not set off on the Ground-line AB, partly to avoid Confusion by describing the Arch of many Circles, partly and chiefly, because it is evident (from the setting off FK and QR) that $EL=EI$ and $TF=TK$.

sent

sent the Side of a Square. Draw through each Point of Division to the principal Point V, as many right Lines or Rays, the two last of which will be VA and VB. Then draw the Ray AD (or BD;) which will cut the foregoing Rays in Points, through which you must draw Parallels to the Ground-line AB. The last of these Parallels will be EF, which will have its Divisions equal between G and H. And the like equal Divisions may be continued from G to E, and from H to F, in order to draw through these new Points of Division, from the principal Point V, other Rays which will upon the Lines DA and DB meet the foregoing Parallels respectively belonging to them. Thus you'll have in Perspective all the squares, which can be comprehended in the Space ABFE. And if you would have more, draw DE, which will cut all the Rays that proceed from V in Points, thro' which (as afore) you must draw Parallels to AB, the last of which will be KL. And so, go on as afore.

27. If you would represent in Perspective a Floor seen fore right, and containing equal Squares seen *Cornerwise*, work thus. The Side AB of the square Floor being determin'd upon the Ground-line AB, describe the Appearance of the square Floor,

To represent
a Floor of
Equal
Squares
seen Corner-
wise.
Fig. 63.

by

by drawing from the two Ends A and B of the Side given, to the principal Point V, the Rays VA and VB; and to the two Points of Distance the Rays DA and DB. These Rays will cut the former in the Points E and F, which being join'd, the Line EF will be parallel to the Ground-line; and ABEF will be the Perspective Square. Having divided AB (or the Side of the Perspective Square) into equal Parts, to each Point of Distance D, draw Rays through each Point of Division; which Rays by their common Intersections, will form the Appearances of the Squares seen Corner-wise. With which Squares it will be easie to fill the Perspective Square ABFE, because all the Rays that proceed from the two Points of Distance, divide equally in the same Points the Side EF parallel to AB; which is also divided equally and in the same Points, by the said Rays which go from the two Points of Distance.

To represent in Perspective a Floor 28.
made of equal Squares seen fore-right, and encompass'd with a Border or Frame, without a geometrical Plane; work thus. Divide the Ground-line AB into as many Parts, alternatively equal and unequal, as you would have Squares and Borders, at the Points 1, 2, 3, 4, 5, 6, 7, 8.
(M) From

To represent a Floor of Squares seen fore-right, and encompassed with a Border.

Fig. 64.

From all these Points draw Rays to V; the first Ray will be VA, the last VB. Then draw the Ray DA, which will cut those that go from V in Points, through which you must draw Parallels to AB; the last of these Parallels will be EF, which terminates the Perspective Square ABFE. And so you will have the Representation of the Floor requir'd. And you may continue it, if you draw another Ray through the Point E and the Point D; and then proceed to work as afore.

29.
To represent a Floor of equal Squares seen Corner-wise, and with a Border, &c.
Fig. 65.

To represent such a Floor as is last mention'd, but seen corner-wise, work thus. Divide the Ground-line AB into Parts as afore, viz, alternatively equal and unequal. Then draw thro' the Points of Division, to the Points of Distance, Rays or Lines; which will (as afore) terminate in the respective Square, this also being to be describ'd as is afore taught.

30.
An useful Way to represent in Perspective one or more Figures.

Lastly, It is observable, that if you describe Squares given in a geometrical Plane, then the Ground-line AB must be divided into Parts equal to the Sides of the given Squares. And the little Perspective Squares, not only will be the Appearances of those in the geometrical Plane, but also may be very useful to represent in Perspective one or more Figures made up of several Lines; as for

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Instance, a fortified Polygon. Namely, the said Polygon (or Wall of a fortified Town) being describ'd on the geometrical Plane with the Squares, it will not be difficult to describe the same likewise in Perspective with the Squares of the Picture, drawing such a Part of it in each Square of the Picture, as is describ'd in the correspondent Square of the geometrical Plane. All which will be sufficiently illustrated by a bare Sight of *Fig. 66.*

And in like Manner, *Fig. 63*, if upon AB a Square be drawn in the geometrical Plane, as ABGH, whose Representation is the perspective Square ABEF; and that Square ABGH on the geometrical Plane be divided into other little Squares by Lines parallel to the Diagonals AG and BH, the said Square ABGH may be made use of, to put or represent in Perspective several Things at once, the Draught of the said Things being describ'd upon the said Square.

And thus I have taken notice of so much of Perspective, as is agreeable to the Design of this Treatise. And therefore shall here put an End to this Treatise of Opticks.

31.

A like Method.

32.

The Conclusion.

F I N I S.

ERRATA to Trigonometry.

PAGE 7. Line 33. For *fit for Use.* Read *not fit for Use.* p. 9. l. 33. f. to ax¹
 $\frac{B-D}{1}$ $\frac{B-D}{2}$
 r. —ax, p. 19. l. 5. f. $\frac{10}{2}$ —=8. r. $\frac{2}{2}$ —=D. p. 30. l. 13. f. CA. r.
 EA. p. 35. l. 22. f. B=350. r. B=35 degr. p. 42. l. 13. f. CA=793. r.
 CA=793. p. 44. l. 6. f. Arch CBA. r. Angle CBA. p. 51. l. ult. f. Line
 BA. r. Line BC. p. 54. l. 16. f. Angles meet. r. Angles, 'till they meet. p.
 55. l. 13. f. at M. r. at N. ibid. l. 19. f. at N. r. at M. p. 56. l. 1. f.
 at the End m. r. at the End n. ibid. l. 4. f. as the End n. r. at the End m.
 p. 65. l. 1. f. so that. r. so shall. p. 82. l. 22. f. to 99. r. to 89.

ERRATA to Mechanicks.

PAGE 6. Line 14. For *contriv'd.* Read *conceiv'd.* p. 10. l. 21. f. little may.
 r. little Body may. p. 13. l. 12. f. Laver. r. Leaver. ibid. l. 26. f.
 but (the. r. but) the. p. 51. l. 21. f. said Squares. r. said Spaces. ibid.
 penult. f. is equidistant from C to Aa. r. is six times as distant from C as is Aa.
 p. 17. l. 1. f. to b. r. to β . ibid. l. 3. f. bC. r. β C. ibid. l. 7. f. (this being.
 r. this being. p. 20. ult. f. G2. r. C2. p. 30. l. 10. f. were. r. were FD.
 ibid. l. 11. f. at C r. at G. ibid. l. 12. f. at r. at B. p. 31. l. 12. f. or
 Mark. r. or Hook. p. 34. l. 23. f. as Fig. r. as Fig. 28, 29, 30. ibid. l. 24.
 f. Pulley) r. Pulley as Fig. 27.) p. 35. l. 19. f. by TS. r. by FS. p. 47. l. 15.
 f. d, e, f. E. By. r. d, e, f. By. p. 50. l. 6. f. CE. r. CG. p. 56. l. 14. f.
 equal. r. equal. p. 57. l. 18. f. EFAB. r. EFHB. p. 65. l. 17. f. How the
 r. Now the.

ERRATA to Opticks.

PAGE 87. Line 14. For Fig. 11. Read Fig. 10. p. 88. ult. f. (Fig.) r.
 (Fig. 28.) p. 89. l. 5, 7, 17, 25. f. (Fig.) r. (Fig. 28.) ibid. l. 14. 27.
 f. (Fig.) r. (Fig. 8) ibid. l. 16. f. Surface BD. r. BC. p. 102. l. 11. f.
 AH. r. AF. p. 107. l. 8. f. is look'd to be. r. is look'd on to be. p. 108. l. 14. f. the
 Convex. r. the Reflex. p. 110. l. 2. f. to the Prop. be requir'd. r. to the Proportion
 here required. p. 113. l. 1. f. Theor. II. r. Theor. IV. ibid. l. penult. f. And when, r.
 And whereas. p. 114. l. 4. f. EDG. r. ADC. p. 116. l. 5. f. hBF. r. hBf. p. 117. l.
 ult. f. 2: hf. r. 4: hf. p. 122. l. 29. subduplicate. r. subduplicate. p. 125. l. 20. f.
 (of the Angle CBM=) r. of the Angle (CBM=). ibid. l. ante penult. f. (as
 Fig.) r. (as Fig. 29) p. 126. l. 10. f. Fig. 30. r. Fig. 36. p. 132. l. 27. f.
 3 AB. r. 2 AB. p. 133. l. 28. f. Concave, r. Convex. ibid. l. 29. f. Convex, r.
 Concave. ibid. l. 30. f. Fig. 36. r. Fig. 35, and 36. p. 141. l. 28. f. Bp. r. Bb.
 p. 148. in Margin, f. Fig. 46. r. Fig. 47. p. 151. l. 3. f. OF r. OE. p. 156.
 l. 8. f. Chrysaline, r. Chrysaline.

In Astronomy, Page 63. Line 15. Read, *Three Diameters of the Moon.*
 In Chronology, Page 33. After these Words, (in the last two Lines
 of the Text,) viz. *and in the Calendar adjoining to this Treatise;* add
 as follows: Or else the Golden Numbers are call'd the Prime, as denoting
 Luna Prima or the First Day of the New Moon; according to which way
 of speaking the Full Moon is frequently stil'd Luna Quarto-decima, as
 falling on the fourteenth Day after the New Moon inclusively.



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Fig. 1.

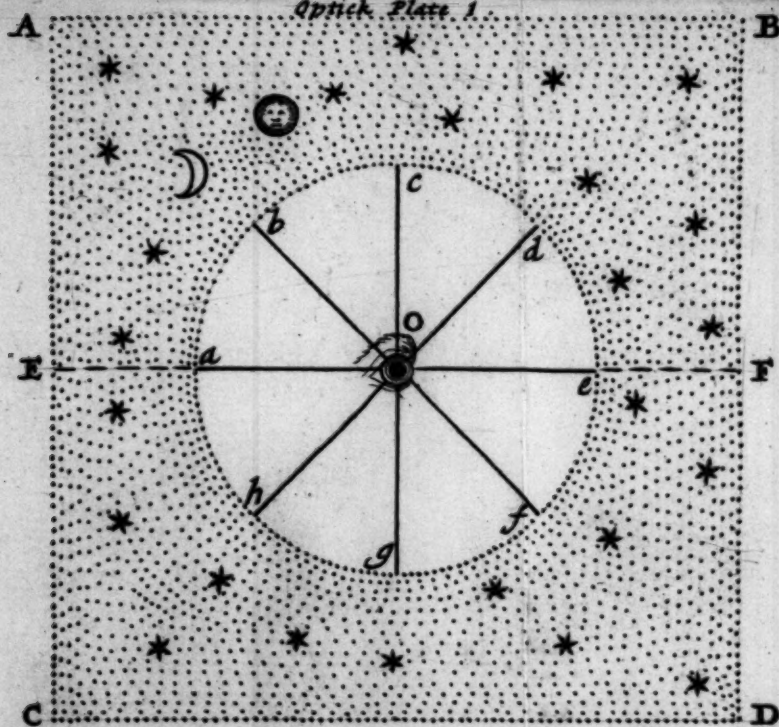


Fig. 2.

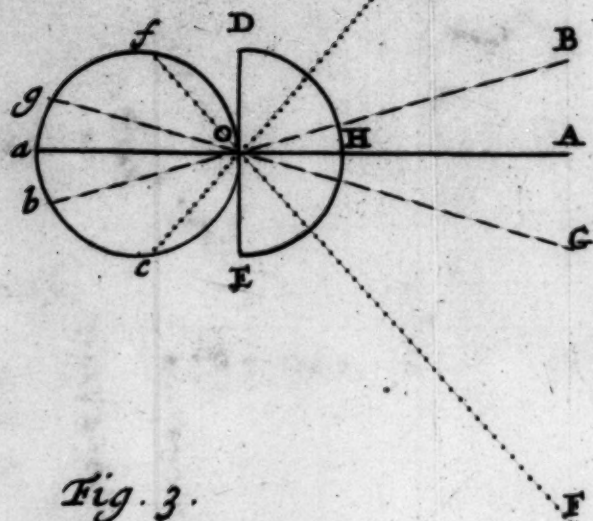
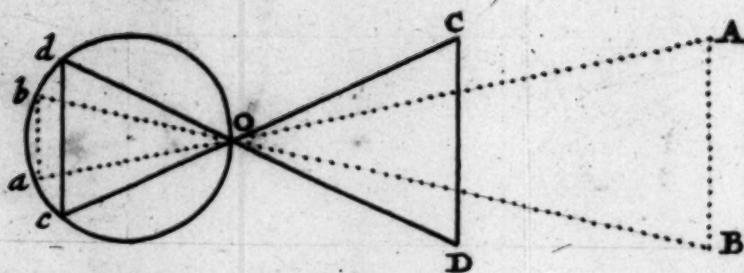


Fig. 3.



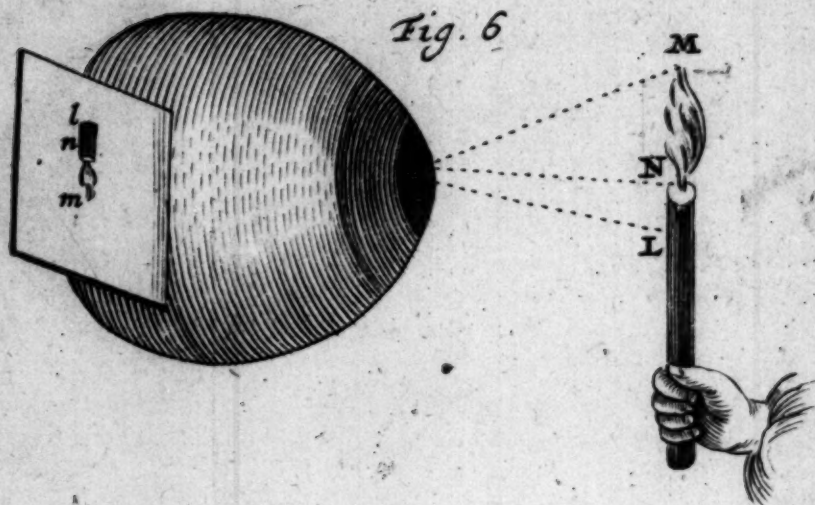
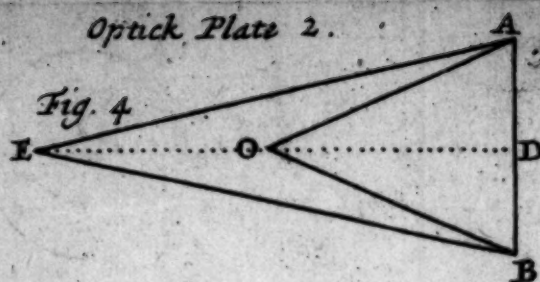
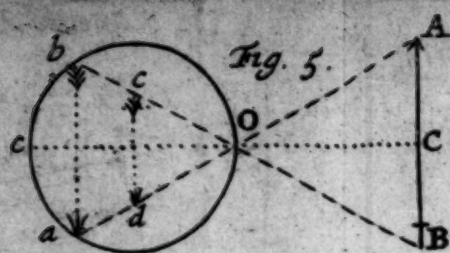
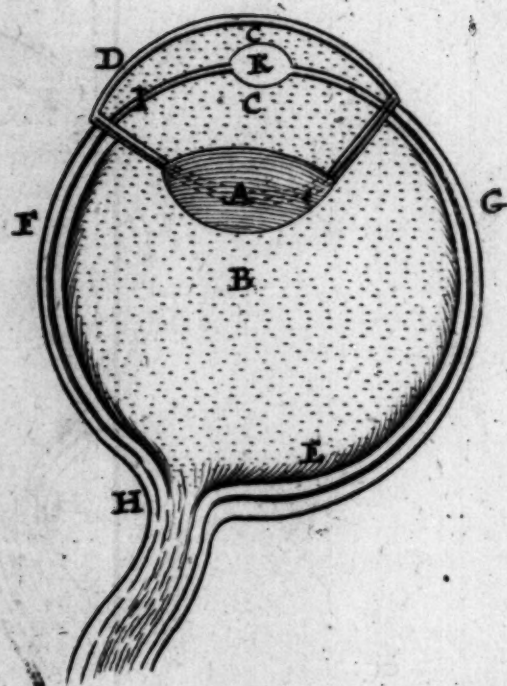


Fig. 7.



Optick Plate 3.

Fig. 8.

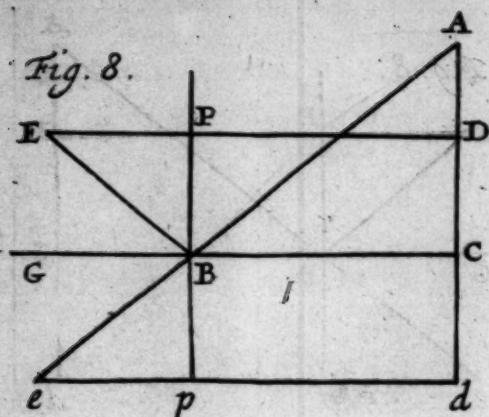


Fig. 10

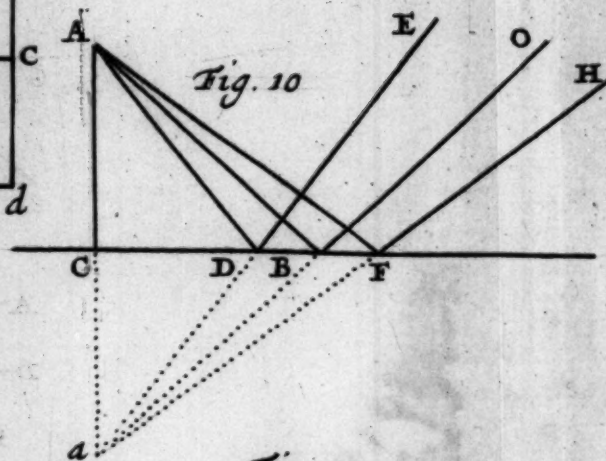


Fig. 9.



Fig. 12

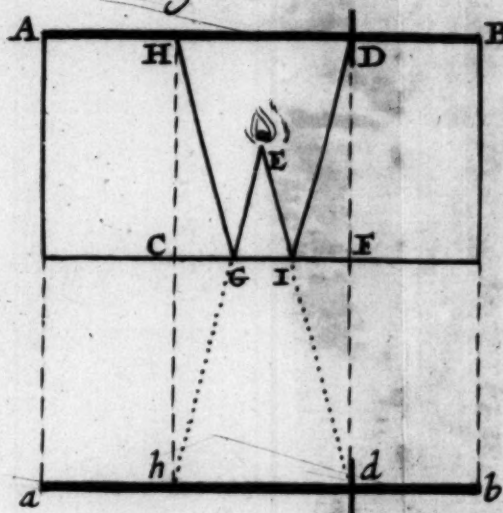


Fig. 11.

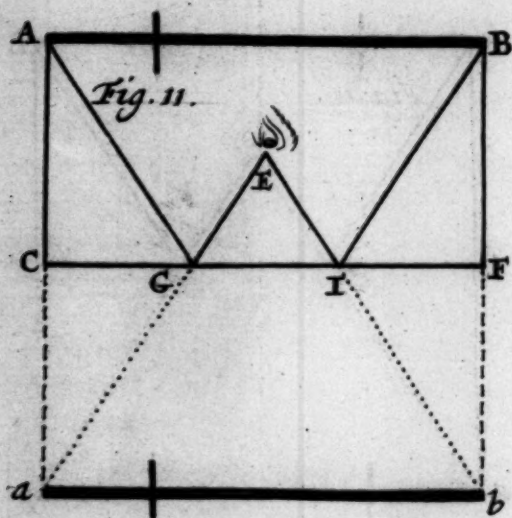


Fig. 13

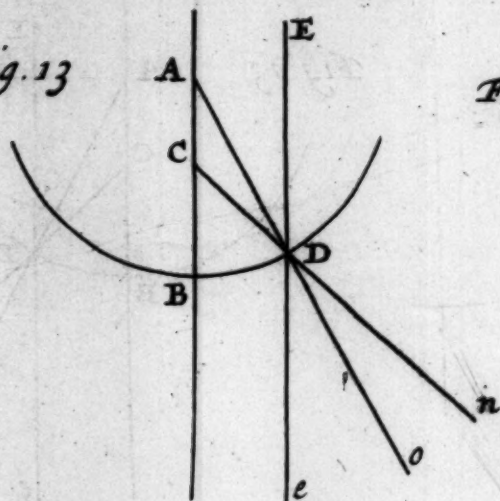
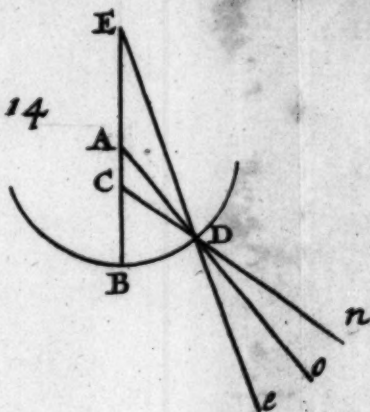


Fig. 14



Optick Plate 4.

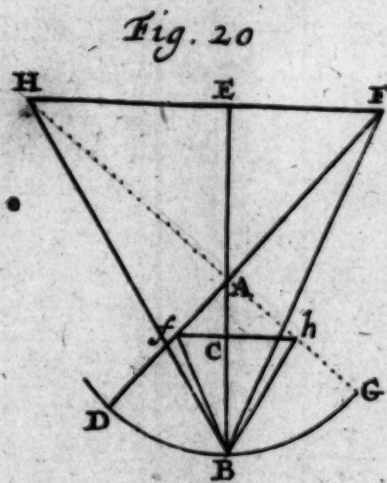
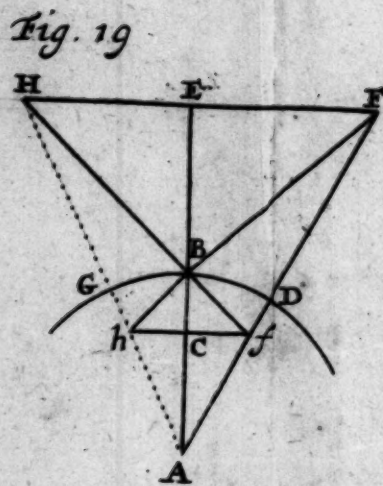
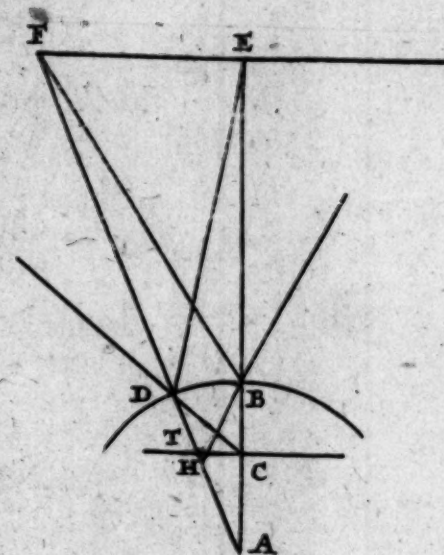
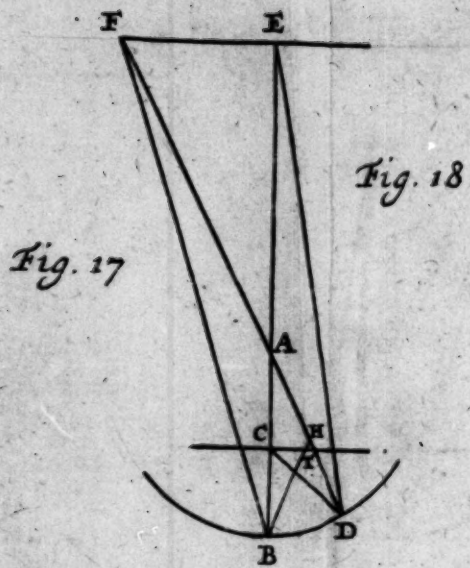
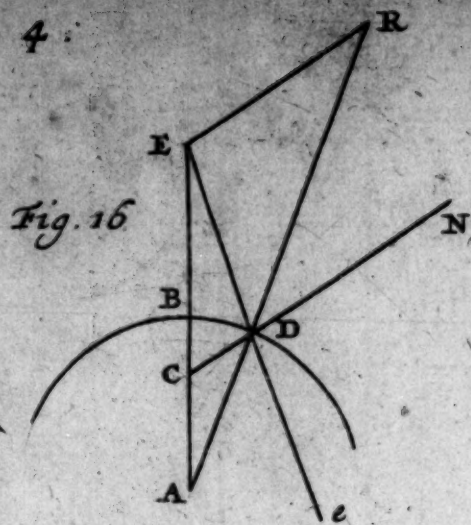
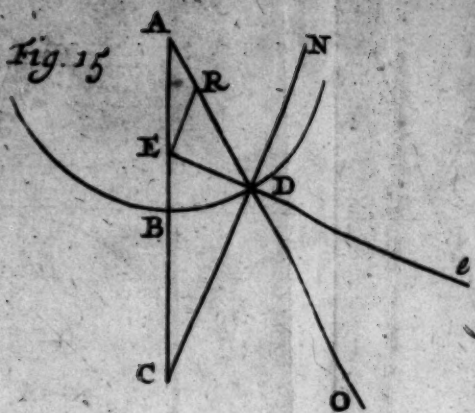


Fig. 21

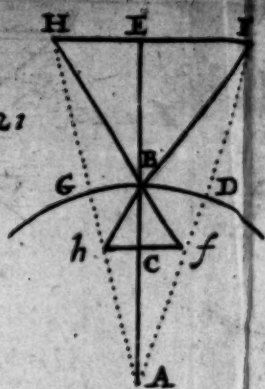
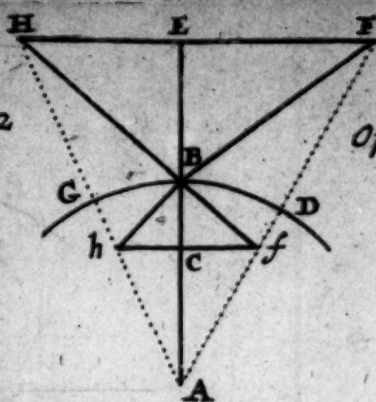


Fig. 22



Optick P. 5.

Fig. 23

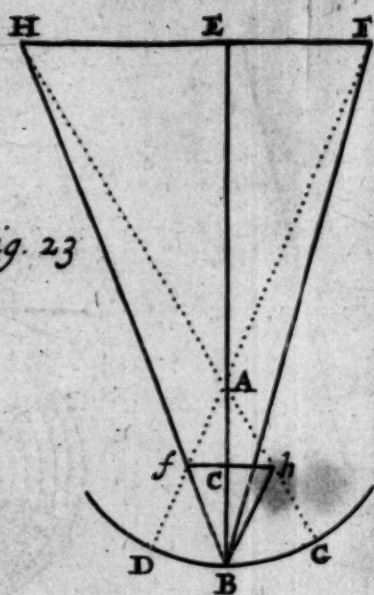


Fig. 26



Fig. 25



Fig. 24



Fig. 27

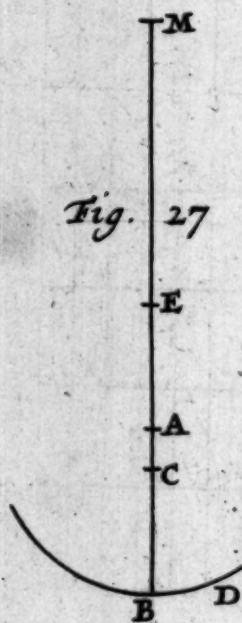
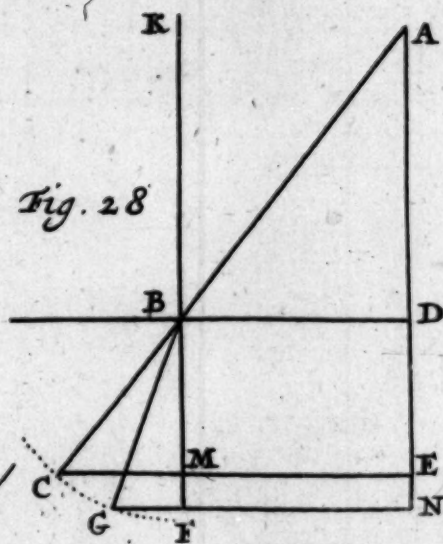


Fig. 28



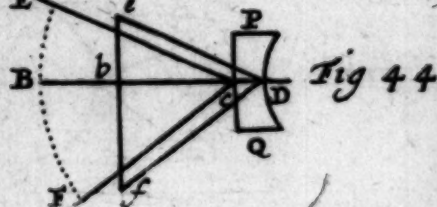
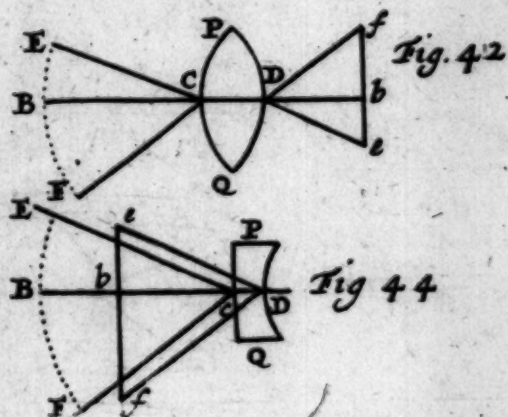
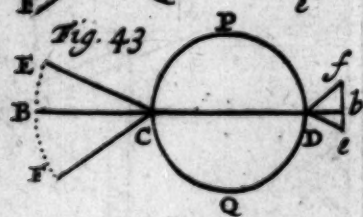
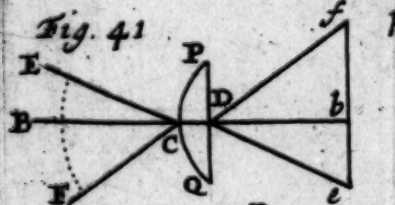
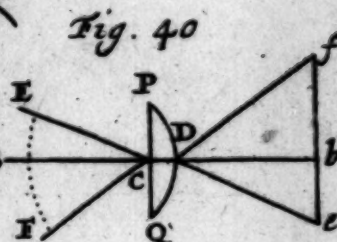
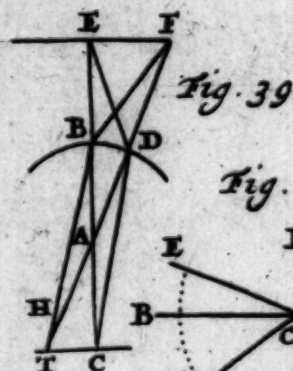
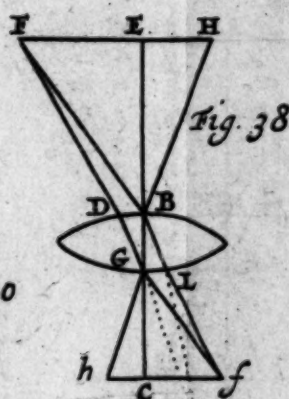
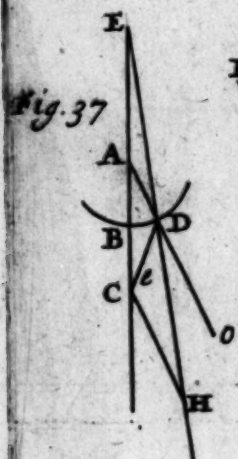
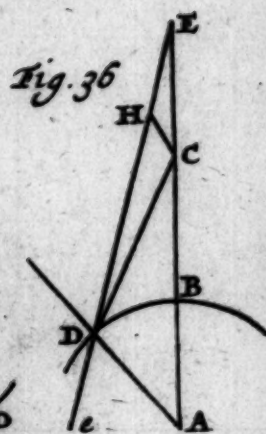
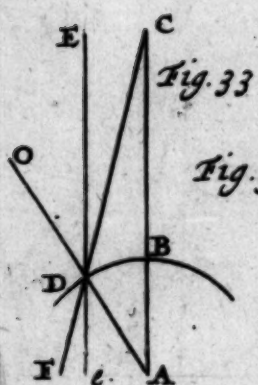
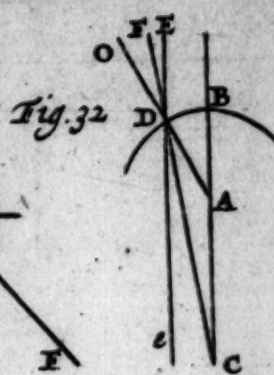
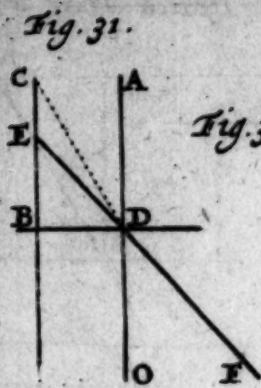
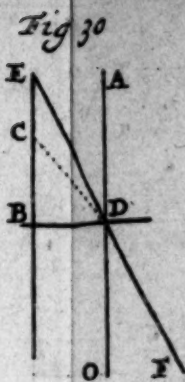
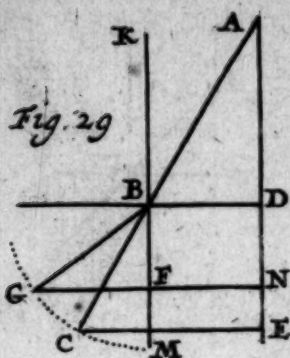


Fig. 45

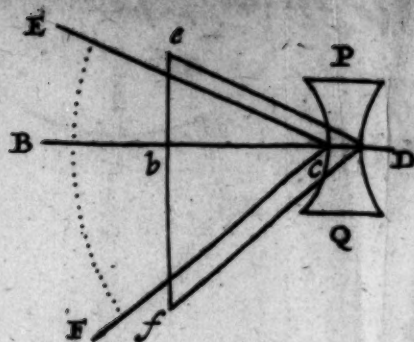


Fig. 46

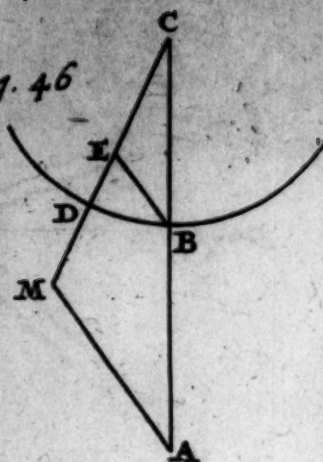


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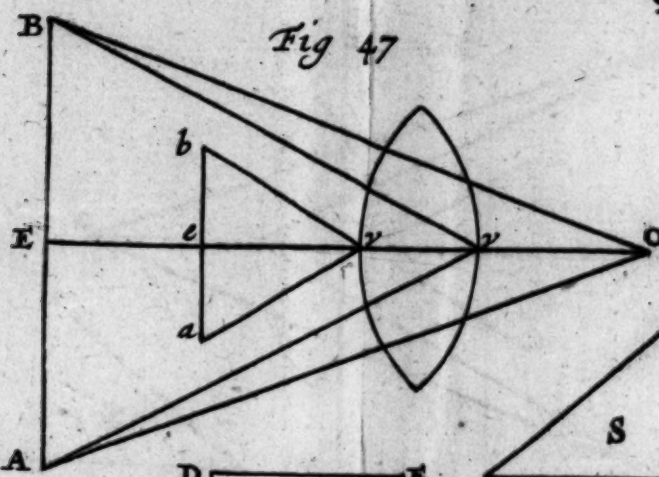


Fig. 47

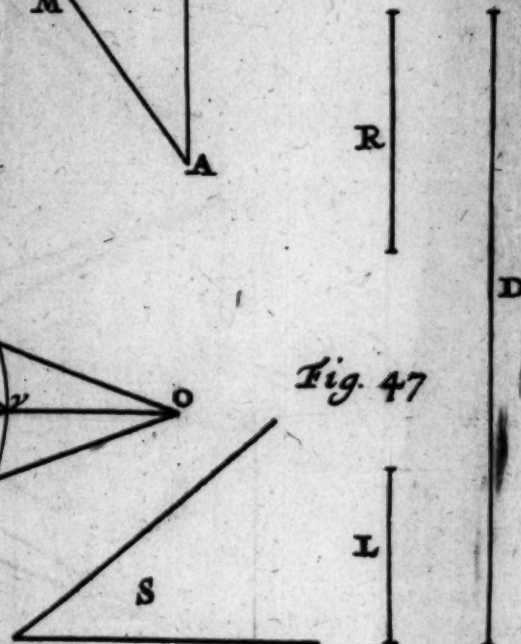


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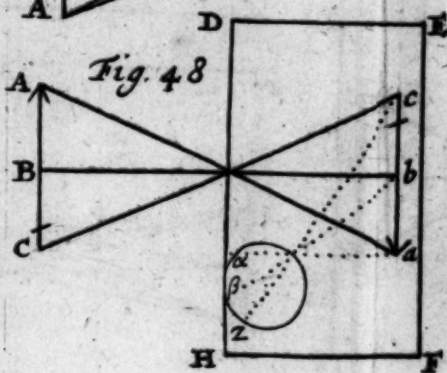


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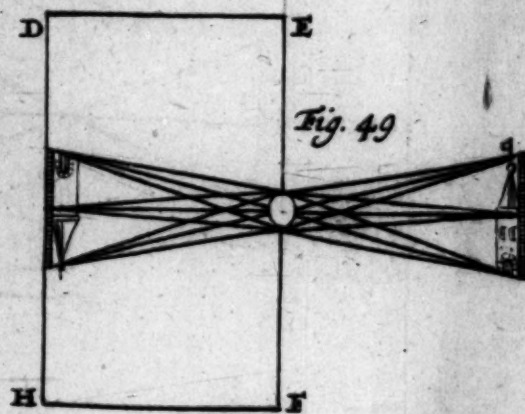


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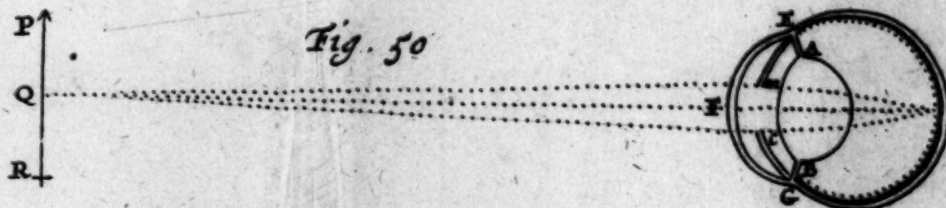


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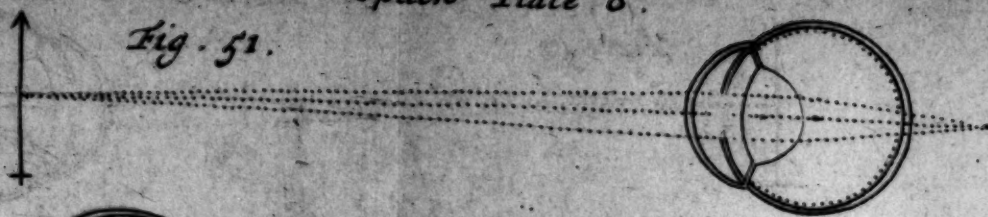


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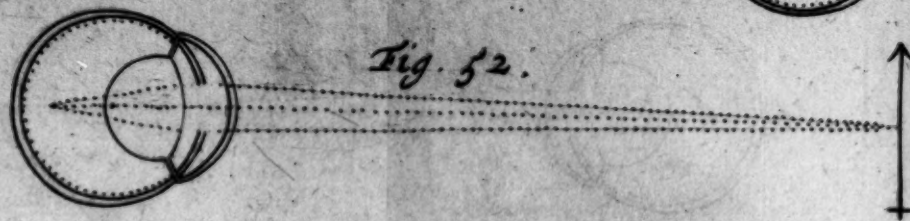


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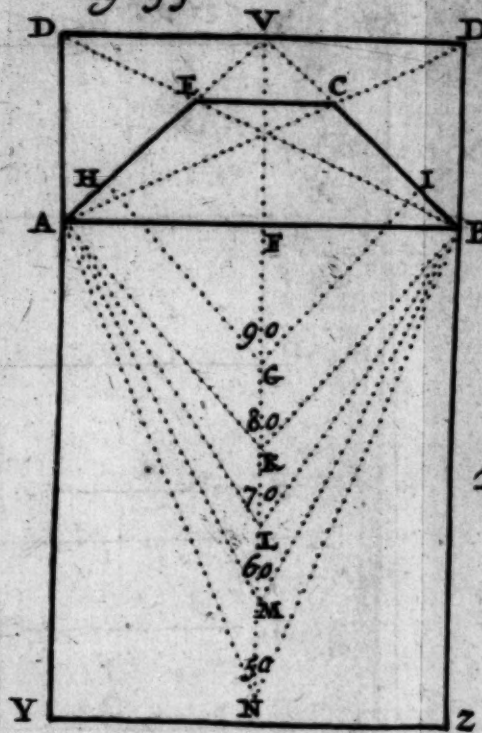


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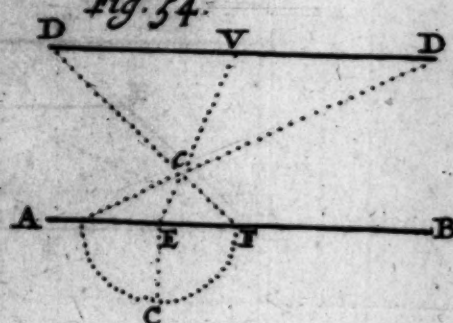


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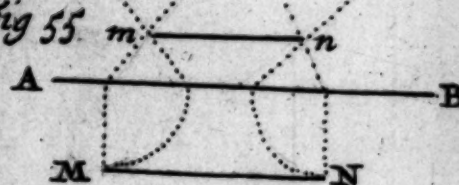


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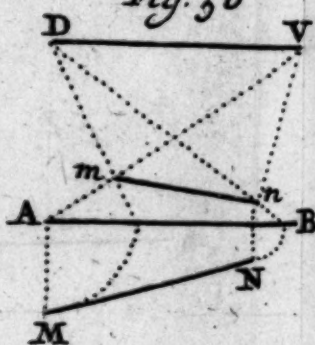


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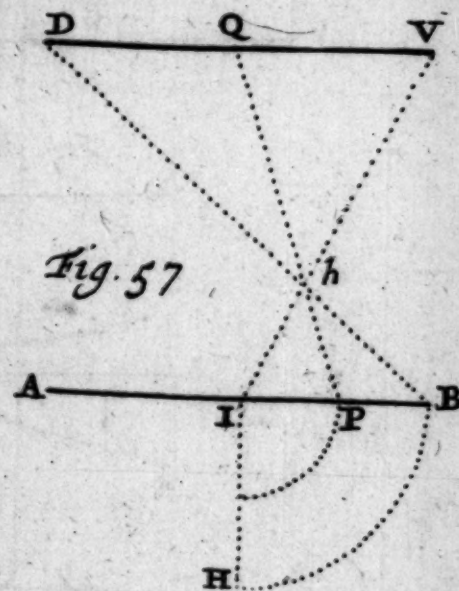


Fig. 58

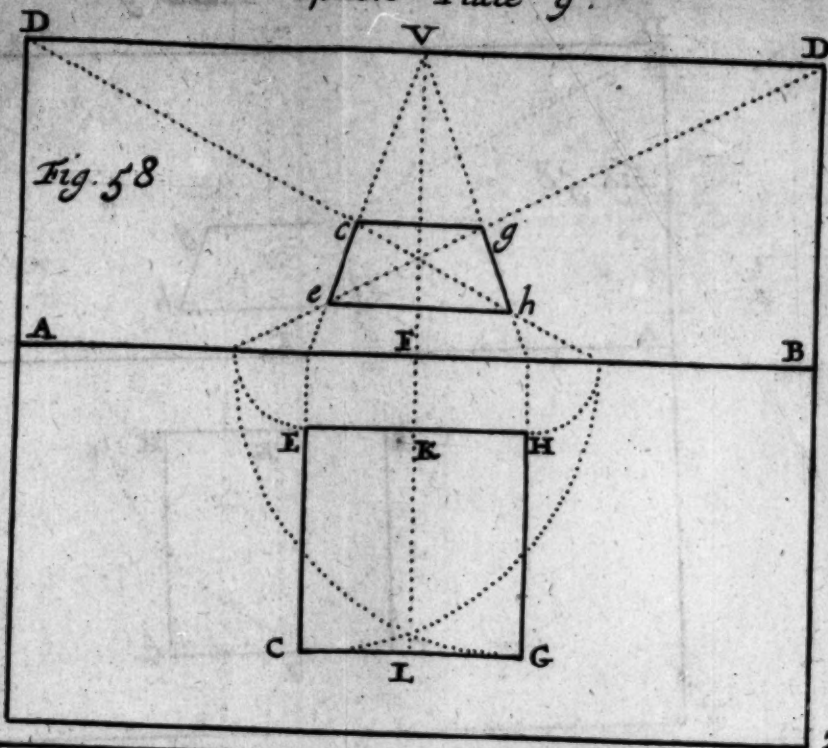


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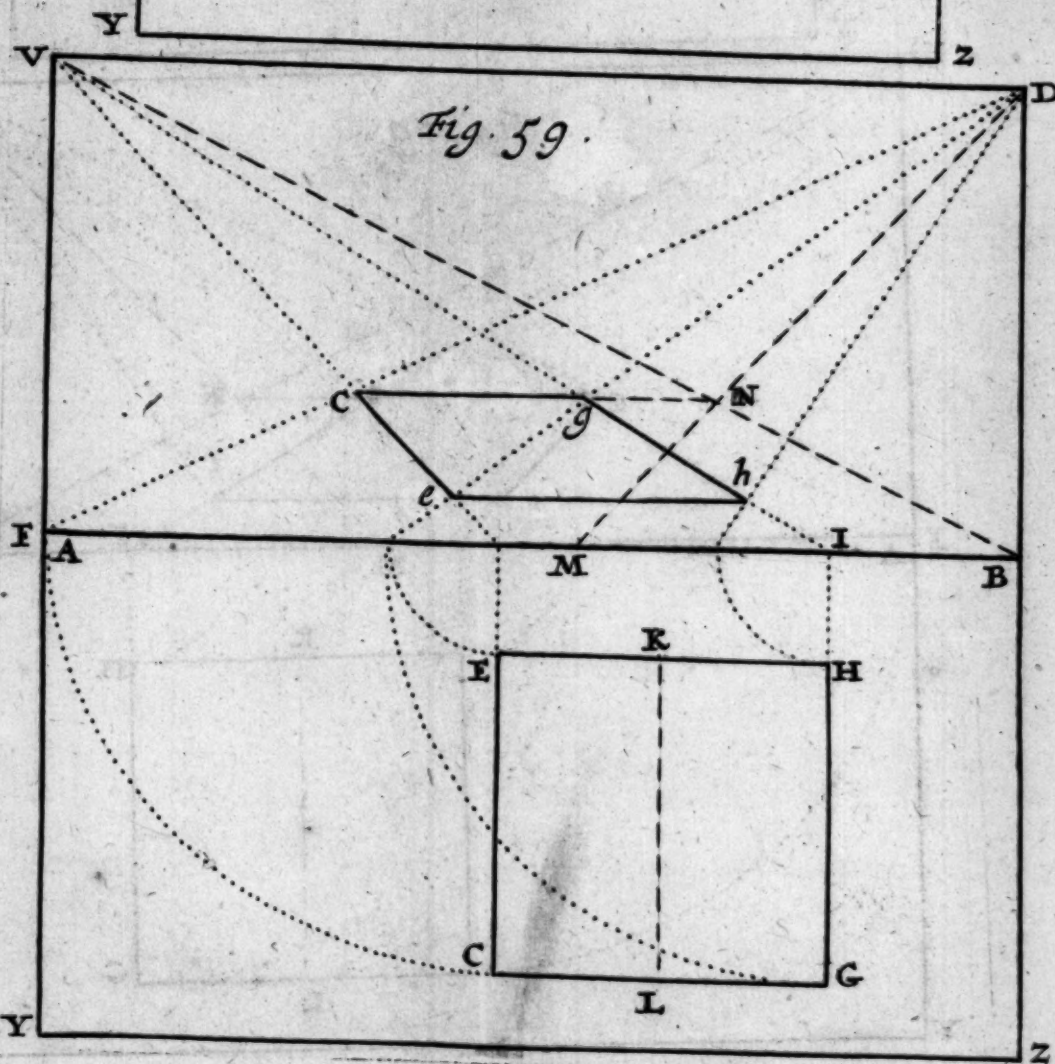


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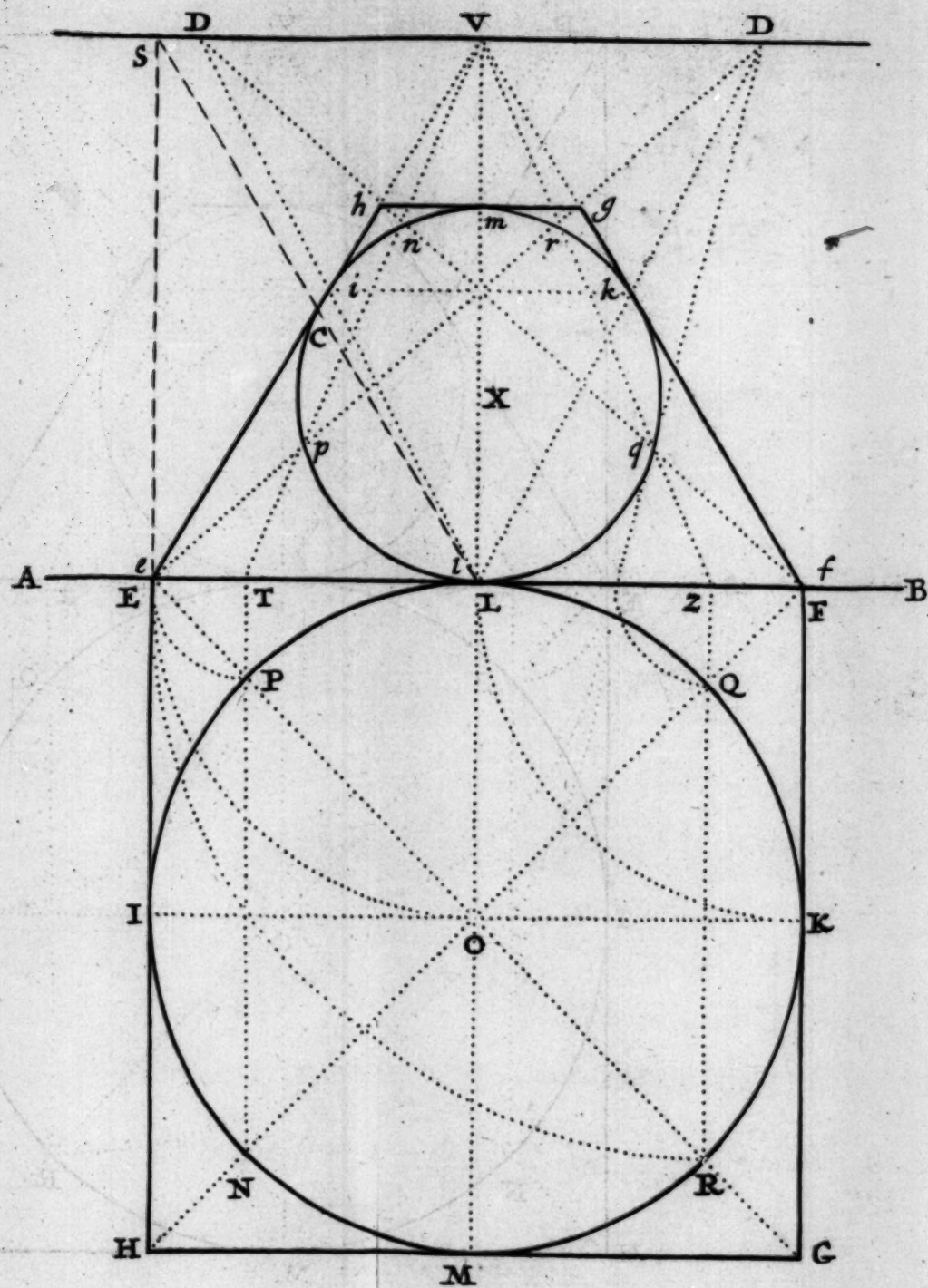


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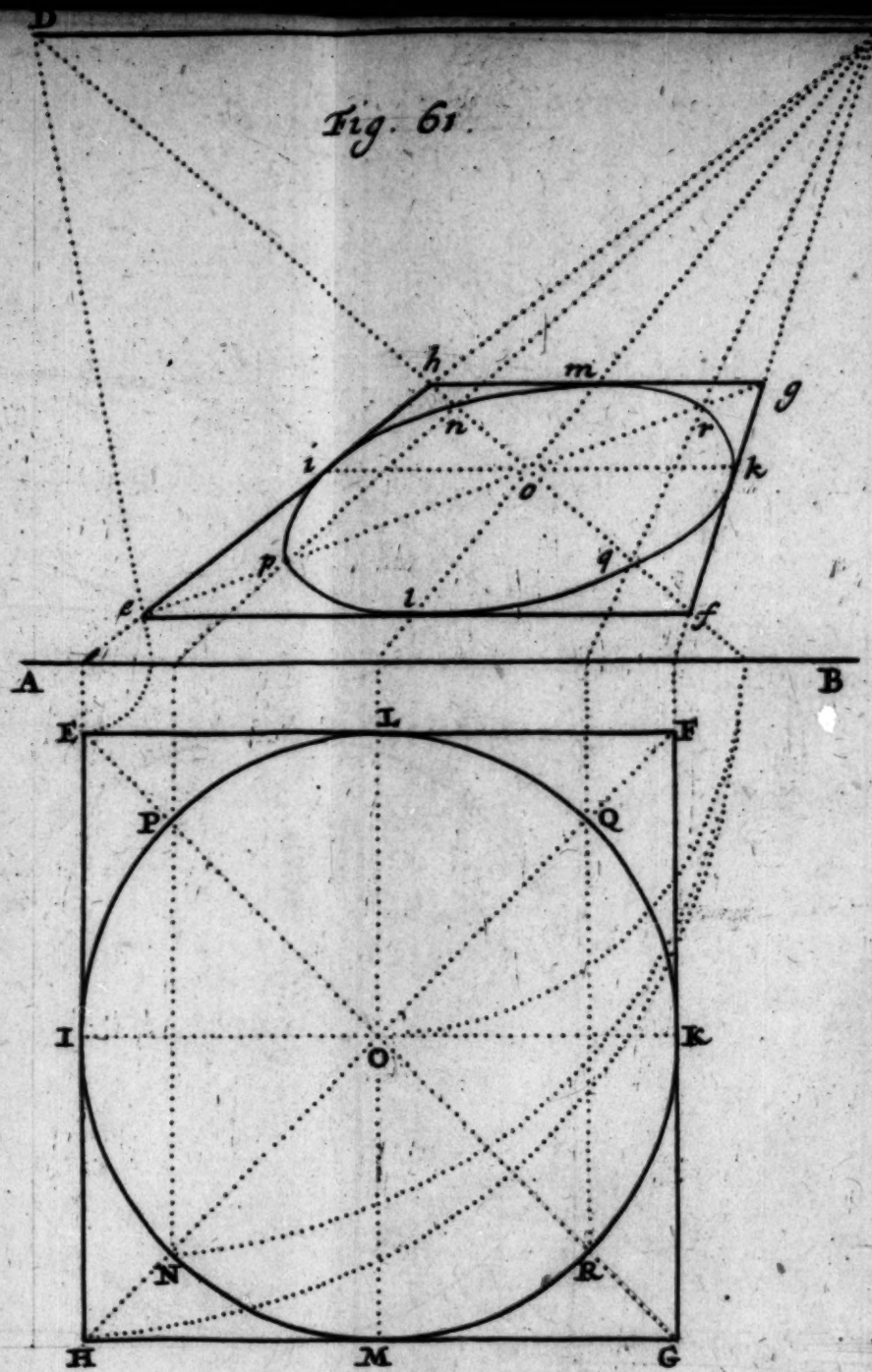


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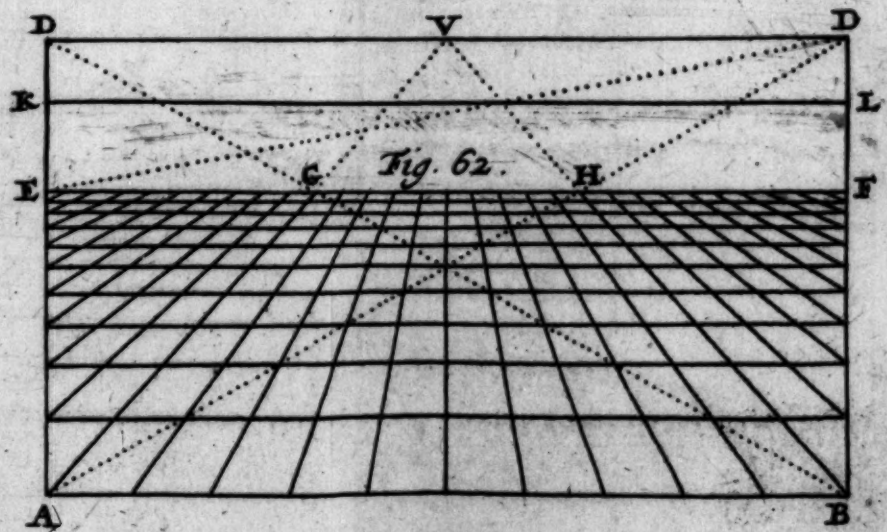


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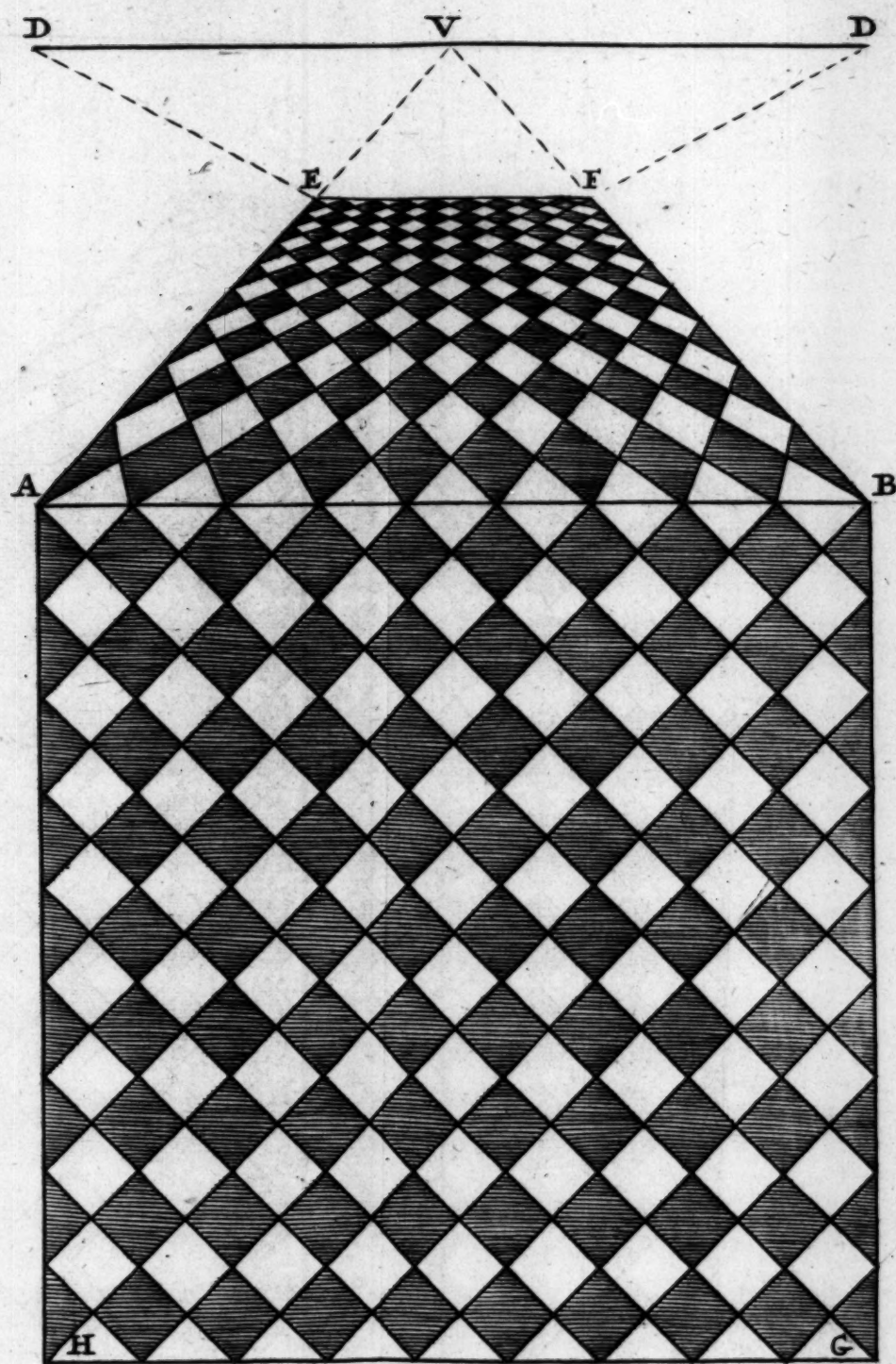


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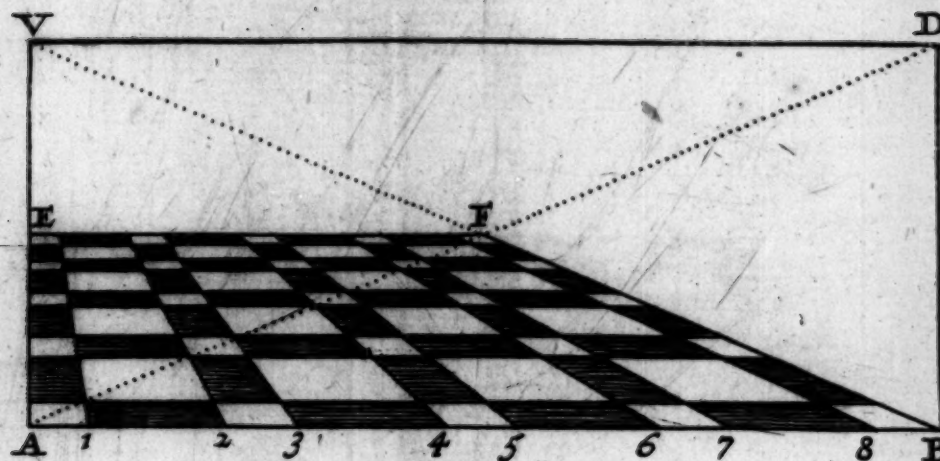


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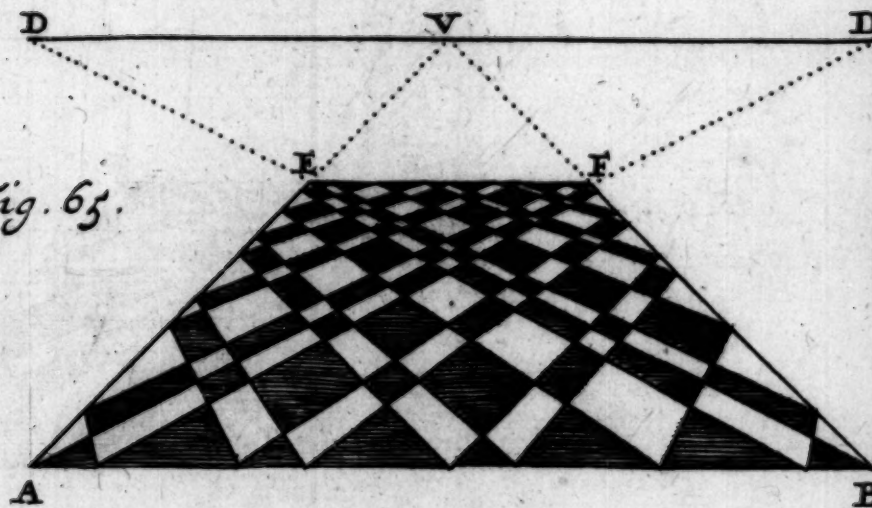


Fig. 66.

